Theorem If X_i are mutually independent noncentral chi-square(δ_i, n_i) random variables for $i = 1, 2, \ldots, m$, then $Y = X_1 + X_2 + \cdots + X_m$ is also a noncentral chi-square random variable.

Proof Let the random variable X_i have the noncentral chi-square distribution with n_i degrees of freedom and noncentrality parameter δ_i with probability density function

$$f_{X_i}(x_i) = \sum_{k=0}^{\infty} \frac{e^{\frac{-\delta_i - x_i}{2}} \left(\frac{\delta_i}{2}\right)^k x_i^{\frac{n_i+2k}{2}-1}}{\left(2^{\frac{n_i+2k}{2}}\right) \Gamma\left(\frac{n_i+2k}{2}\right) k!} \qquad x > 0$$

for $i = 1, 2, \ldots, m$. The moment generating function for X_i is

$$M_{X_i}(t) = \frac{e^{\delta_i t/(1-2t)}}{(1-2t)^{n_i/2}} \qquad t < 1/2$$

for $i = 1, 2, \ldots, m$. Let the random variable $Y = \sum_{i=1}^{m} X_i$. The moment generating function of Y is

$$\begin{aligned}
E\left[e^{tY}\right] &= E\left[e^{t\left(\sum_{i=1}^{m} X_i\right)}\right] \\
&= E\left[e^{tX_1} e^{tX_2} \ldots e^{tX_m}\right] \\
&= E\left[e^{tX_1}\right] E\left[e^{tX_2}\right] \ldots E\left[e^{tX_m}\right] \\
&= \frac{e^{\delta_1 t/(1-2t)}}{(1-2t)^{n_1/2}} \cdot \frac{e^{\delta_2 t/(1-2t)}}{(1-2t)^{n_2/2}} \cdots \frac{e^{\delta_n t/(1-2t)}}{(1-2t)^{n_m/2}} \\
&= \frac{e^{\sum_{i=1}^{m} \delta_i t/(1-2t)}}{(1-2t)^{\sum_{i=1}^{m} n_i/2}} \\
&= \frac{e^{t \sum_{i=1}^{m} \delta_i/(1-2t)}}{(1-2t)^{\sum_{i=1}^{m} n_i/2}} \qquad t < 1/2,
\end{aligned}$$

which is the moment generating function of a noncentral chi-square random variable with $\sum_{i=1}^{m} n_i$ degrees of freedom and noncentrality parameter $\sum_{i=1}^{m} \delta_i$.