Theorem The limiting distribution of a negative hypergeometric random variable is a binomial random variable as $n_3 \to \infty$, $n_1 \to \infty$, with $p = n_1/n_3$ and $n_2 = n$.

Proof Let X be a negative hypergeometric random variable. Then, the limiting probability density function can be found by taking the limits:

$$
\begin{aligned}
&\lim_{n_3\to\infty}\lim_{n_1\to\infty} f(x)\\
&= \lim_{n_3\to\infty}\lim_{n_1\to\infty} \binom{n_1+x-1}{x}\binom{n_3-n_1+n_2-x-1}{n_2-x}\Big/\binom{n_3+n_2-1}{n_2}\\
&= \lim_{n_3\to\infty}\lim_{n_1\to\infty} \binom{pn_3+x-1}{x}\binom{n_3-pn_3+n-x-1}{n-x}\Big/\binom{n_3+n-1}{n}\\
&= \lim_{n_3\to\infty}\lim_{n_1\to\infty} \frac{\frac{(pn_3+x-1)!}{x!(pn_3-1)!}\cdot\frac{(n_3-pn_3+n-x-1)!}{(n_3-pn_3-1)!(n-x)!}}{\frac{(n_3+n-1)!}{n!(n_3-1)!}}\\
&= \lim_{n_3\to\infty}\lim_{n_1\to\infty} \binom{n}{x}\frac{(pn_3+x-1)\ldots(pn_3+x-x)}{(n_3+n-1)\ldots(n_3+n-n)}(n_3(1-p)+n-x-1)\ldots n_3(1-p)\\
&= \lim_{n_3\to\infty}\lim_{n_1\to\infty} \binom{n}{x}p^x(1-p)^{n-x}\frac{(n_3+x/p-1/p)\ldots(n_3)(n_3-\frac{n}{1-p}-\frac{x}{1-p})\ldots(n_3)}{(n_3+n-1)\ldots(n_3+n-n)}\\
&= \binom{n}{x}p^x(1-p)^{n-x} \qquad\qquad x = 0, 1, 2, \ldots, n.
\end{aligned}
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The last step can take place for p converges as both n_3 and n_1 go to infinity. Also, there are x and $x-n$ terms going to infinity in the numerator, while there are n terms going to infinity in the denominator. These terms go to 1 as you take the limit.