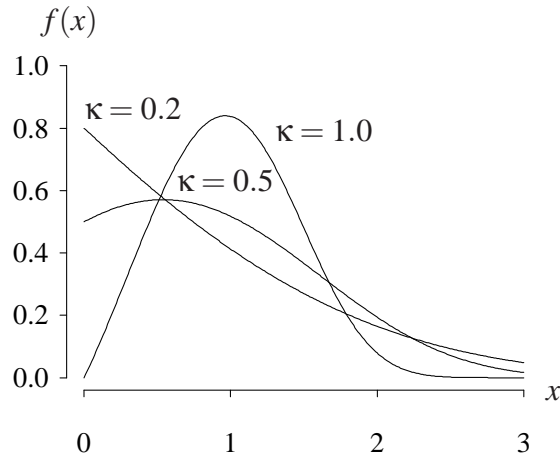


Muth distribution (from <http://www.math.wm.edu/~leemis/chart/UDR/UDR.html>)

The shorthand $X \sim \text{Muth}(\kappa)$ is used to indicate that the random variable X has the Muth distribution with parameter κ . A Muth random variable X with parameter κ has probability density function

$$f(x) = (e^{\kappa x} - \kappa)e^{-\frac{e^{\kappa x}}{\kappa} + \kappa x + \frac{1}{\kappa}} \quad x > 0$$

for $0 < \kappa \leq 1$. The probability density function for three different values of κ is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \leq x) = 1 - e^{-\frac{e^{\kappa x}}{\kappa} + \kappa x + \frac{1}{\kappa}} \quad x > 0.$$

The survivor function on the support of X is

$$S(x) = P(X \geq x) = e^{-\frac{e^{\kappa x}}{\kappa} + \kappa x + \frac{1}{\kappa}} \quad x > 0.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = e^{\kappa x} - \kappa \quad x > 0.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = \frac{e^{\kappa x}}{\kappa} - \kappa x - \frac{1}{\kappa} \quad x > 0.$$

The inverse distribution function, median, moment generating function and characteristic function of X are not mathematically tractable.

The population mean is

$$E[X] = 1.$$

APPL verification: The APPL statements

```
X := MuthRV(kappa);  
CDF(X);  
SF(X);  
HF(X);  
CHF(X);  
Mean(X);
```

verify the verify the cumulative distribution, survivor function, hazard function, cumulative hazard function, population mean.