

**Theorem** Random variates from the minimax distribution with parameters  $\gamma$  and  $\beta$  can be generated in closed-form by inversion.

**Proof** The  $\text{minimax}(\beta, \gamma)$  distribution has probability density function

$$f(x) = \beta\gamma x^{\beta-1}(1-x^\beta)^{\gamma-1} \quad 0 < x < 1$$

and cumulative distribution function

$$F(x) = 1 - (1-x^\beta)^\gamma \quad 0 < x < 1.$$

Equating the cumulative distribution function to  $u$ , where  $0 < u < 1$  yields an inverse cumulative distribution function

$$F^{-1}(u) = (1 - (1-u)^{1/\gamma})^{1/\beta} \quad 0 < u < 1.$$

So a closed-form variate generation algorithm using inversion for the  $\text{minimax}(\beta, \gamma)$  distribution is

```
generate  $U \sim U(0, 1)$   
 $X \leftarrow (1 - (1 - u)^{1/\gamma})^{1/\beta}$   
return( $X$ )
```

**APPL verification:** The APPL statements

```
X := [[x -> 1 - (1 - x ^ beta) ^ gamma], [0, 1], ["Continuous", "CDF"]];  
IDF(X);
```

verify the inverse distribution function of a minimax random variable.