

Theorem Let $X_i \sim \text{minimax}(\beta, \gamma_i)$ for $i = 1, 2, \dots, n$ be mutually independent random variables. The minimum of X_1, X_2, \dots, X_n is also a minimax random variable with parameters β and $\sum_{i=1}^n \gamma_i$.

Proof The cumulative distribution function of a $\text{minimax}(\beta, \gamma)$ random variable X is given by

$$F_X(x) = 1 - (1 - x^\beta)^\gamma \quad 0 < x < 1.$$

The cumulative distribution function of $Y = \min\{X_1, X_2, \dots, X_n\}$ is

$$\begin{aligned} F_Y(y) &= P(Y \leq y) \\ &= 1 - P(Y \geq y) \\ &= 1 - P(\min\{X_1, X_2, \dots, X_n\} \geq y) \\ &= 1 - P(X_1 \geq y, X_2 \geq y, \dots, X_n \geq y) \\ &= 1 - P(X_1 \geq y) P(X_2 \geq y) \dots P(X_n \geq y) \\ &= 1 - (1 - y^\beta)^{\gamma_1} (1 - y^\beta)^{\gamma_2} \dots (1 - y^\beta)^{\gamma_n} \\ &= 1 - (1 - y^\beta)^{\sum_{i=1}^n \gamma_i} \quad 0 < y < 1, \end{aligned}$$

which is the cumulative distribution function of a $\text{minimax}(\beta, \sum_{i=1}^n \gamma_i)$ random variable.