Theorem The Gompertz(κ, δ) distribution is a special case of the Makeham(δ, κ, γ) distribution when $\gamma = 0$.

Proof The Makeham distribution has probability density function

$$f(x) = (\gamma + \delta\kappa^x)e^{-\gamma x - \frac{\delta(\kappa^x - 1)}{\ln \kappa}} \qquad x > 0.$$

Substituting $\gamma = 0$ yields

$$f(x) = (0 + \delta\kappa^x)e^{0 - \frac{\delta(\kappa^x - 1)}{\ln \kappa}} = \delta\kappa^x e^{-\frac{\delta(\kappa^x - 1)}{\ln \kappa}} \qquad x > 0$$

which is the probability density function of a Gompertz distribution.

APPL verification: The APPL statements

```
X := MakehamRV(gam, d, k);
subs(gam = 0, X[1][1](x));
Y := GompertzRV(d, k);
```

yield identical functional forms:

$$f(x) = \delta\kappa^x e^{-\frac{\delta(\kappa^x - 1)}{\ln \kappa}} \qquad x > 0,$$

so the Gompertz(κ, δ) distribution is a special case of the Makeham(δ, κ, γ) distribution when $\gamma = 0$.