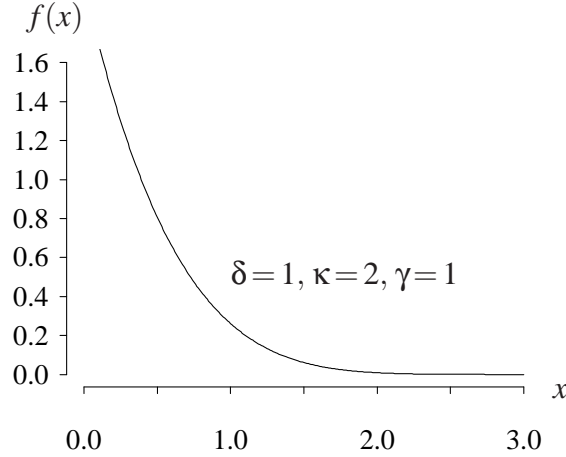


Makeham distribution (from <http://www.math.wm.edu/~leemis/chart/UDR/UDR.html>)

The shorthand $X \sim \text{Makeham}(\delta, \kappa, \gamma)$ is used to indicate that the random variable X has the Makeham distribution with parameters δ , κ , γ . A Makeham random variable X with parameters δ , κ , and γ has probability density function

$$f(x) = (\gamma + \delta \kappa^x) e^{-\gamma x - \delta(\kappa^x - 1)/\ln(\kappa)} \quad x > 0,$$

for all $\delta > 0$, $\kappa > 1$, and $\gamma > 0$. The Makeham distribution is used to model adult lifetimes by actuaries. The probability density function with $\delta = 1$, $\kappa = 2$, and $\gamma = 1$ is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \leq x) = 1 - e^{-(\gamma x \ln(\kappa) + \delta \kappa^x - \delta)/\ln(\kappa)} \quad x > 0.$$

The survivor function on the support of X is

$$S(x) = P(X \geq x) = e^{-(\gamma x \ln(\kappa) + \delta \kappa^x - \delta)/\ln(\kappa)} \quad x > 0.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \gamma + \delta \kappa^x \quad x > 0.$$

The cumulative hazard function on the support of X is mathematically intractable.

$$H(x) = \frac{\gamma x \ln(\kappa) + \delta \kappa^x - \delta}{\ln(\kappa)} \quad x > 0.$$

The inverse distribution function, moment generating function and characteristic function of X are mathematically intractable. The population mean, variance, skewness, and kurtosis of X are also mathematically intractable.

APPL verification: The APPL statements

```
assume(delta > 0);
assume(gamma > 0);
assume(kappa > 1);
X := [[x-> (y+delta*kappa^x)*exp(-y*x - delta*(kappa^x - 1)/ln(kappa))],
      [0,infinity],[ "Continuous", "PDF" ]];
CDF(X);
SF(X);
HF(X);
CHF(X);
```

verify the cumulative distribution function, survivor function, hazard function, and cumulative hazard function.