Theorem The Lomax distribution has the variate generation property. That is, the inverse cumulative distribution function of a Lomax(\(\lambda, \kappa\)) random variable can be expressed in closed-form.

Proof The cumulative distribution function of a Lomax random variable \(X\) on its support is given by

\[
F(x) = \int_0^x \frac{\lambda \kappa}{(1 + \lambda t)^{\kappa+1}} dt
= \left[-\frac{1}{(1 + \lambda t)^\kappa}\right]_0^x
= 1 - \frac{1}{(1 + \lambda x)^\kappa} \quad x > 0.
\]

Now we find the inverse cumulative distribution function \(F^{-1}(u)\) by solving

\[
u = 1 - \frac{1}{(1 + \lambda x)^\kappa}
\]

for \(x\) yielding

\[
F^{-1}(u) = \frac{(1 - u)^{-1/\kappa} - 1}{\lambda} \quad 0 < u < 1.
\]

Therefore, the Lomax distribution has the variate generation property.

APPL verification: The APPL statements

\[
X := \text{LomaxRV(kappa, lambda)};
\]
\[
\text{CDF(X)};
\]
\[
\text{IDF(X)};
\]

confirm the inverse cumulative distribution function given above.