Theorem The Lomax distribution has the variate generation property. That is, the inverse cumulative distribution function of a Lomax(λ, κ) random variable can be expressed in closed-form.

Proof The cumulative distribution function of a Lomax random variable X on its support is given by

$$\begin{aligned} F(x) &= \int_0^x \frac{\lambda\kappa}{(1+\lambda t)^{\kappa+1}}\, dt \\ &= \left[-\frac{1}{(1+\lambda t)^{\kappa}}\right]_0^x \\ &= 1 - \frac{1}{(1+\lambda x)^{\kappa}} \qquad\qquad x > 0. \end{aligned}$$

Now we find the inverse cumulative distribution function $F^{-1}(u)$ by solving

$$u = 1 - \frac{1}{(1+\lambda x)^{\kappa}}$$

for x yielding

$$F^{-1}(u) = \frac{(1-u)^{-1/\kappa} - 1}{\lambda} \qquad\qquad 0 < u < 1.$$

Therefore, the Lomax distribution has the variate generation property.

APPL verification: The APPL statements

```
X := LomaxRV(kappa, lambda);
CDF(X);
IDF(X);
```

confirm the inverse cumulative distribution function given above.