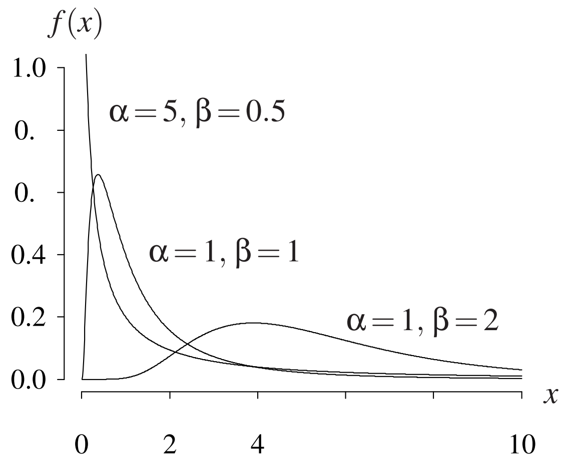
Log normal distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim$ log normal(α, β) is used to indicate that the random variable X has the log normal distribution with parameters α and β. A log normal random variable X with parameters α and β has probability density function

$$f(x) = \frac{1}{x\beta\sqrt{2\pi}} e^{-\frac{1}{2}(\ln(x/\alpha)/\beta)^2} \qquad x > 0$$

for α and $\beta > 0$. The log normal distribution can be used to model the lifetime of an object, the weight of a person, or a service time. The central limit theorem indicates that the log normal distribution is useful for modeling random variables that can be thought of as a product of several independent random variables. The probability density function with three different parameter settings is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \frac{1}{2} + \frac{1}{2}\,\mathrm{erf}\left(\frac{\sqrt{2}\,(\ln(x) - \alpha)}{2\beta}\right) \qquad x > 0,$$

where

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = \frac{1}{2} - \frac{1}{2}\,\mathrm{erf}\left(\frac{\sqrt{2}\,(\ln(x) - \alpha)}{2\beta}\right) \qquad x > 0.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = -\sqrt{2}e^{-(\ln(x)-\alpha)^2/2\beta^2} \frac{1}{\sqrt{\pi}} x^{-1}\beta^{-1} \left(-1 + \mathrm{erf}\left(\frac{\sqrt{2}\,(\ln(x) - \alpha)}{2\beta}\right)\right)^{-1} \qquad x > 0.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = \ln(2) + i\pi - \ln\left(-1 + \operatorname{erf}\left(\frac{\sqrt{2}\,(\ln(x) - \alpha)}{2\beta}\right)\right) \qquad x > 0.$$

The inverse distribution function, moment generating function, and characteristic function of X are mathematically intractable.

The median of X is α.

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \alpha e^{\beta^2/2} \qquad V[X] = \alpha^2 e^{\beta^2}(e^{\beta^2} - 1)$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = (e^{\beta^2} + 2)(e^{\beta^2} - 1)^{1/2} \qquad E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = e^{4\beta^2} + 2e^{3\beta^2} + 3e^{2\beta^2} - 3$$

APPL verification: The APPL statements

```
assume(alpha > 0);
assume(beta > 0);
X := [[x-> (1/(x*beta*sqrt(2*Pi)))*exp((-1/2)*(ln(x/alpha)/beta)^2)],
     [0,infinity],["Continuous","PDF"]];
CDF(X);
SF(X);
HF(X);
CHF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the cumulative distribution function, survivor function, hazard function, cumulative hazard function, population mean, variance, skewness, and kurtosis.