Theorem The log logistic distribution has the variate generation property. That is, the inverse cumulative distribution function of a log logistic($\lambda, \kappa$) random variable can be expressed in closed-form.

Proof The probability density function of a log logistic($\lambda, \kappa$) random variable is

$$f(x) = \frac{\lambda \kappa (\lambda x)^{\kappa-1}}{(1 + (\lambda x)^\kappa)^2} \quad x > 0.$$ 

The cumulative distribution function is

$$F(x) = \frac{(\lambda x)^\kappa}{1 + (\lambda x)^\kappa} \quad x > 0.$$ 

Equating the cumulative distribution function to $u$ where $0 < u < 1$ yields the inverse cumulative distribution function

$$F^{-1}(u) = \left(\frac{u}{\lambda^\kappa (1 - u)}\right)^{1/\kappa} \quad 0 < u < 1.$$ 

So a closed form variate generation algorithm for the log logistic distribution is

\begin{align*}
\text{generate } U &\sim U(0, 1) \\
X &\leftarrow (u / (\lambda^\kappa (1 - u)))^{1/\kappa} \\
\text{return}(X)
\end{align*}

APPL verification: The APPL statements

\begin{verbatim}
X := LogLogisticRV(lambda, kappa); 
CDF(X); 
IDF(X); 
\end{verbatim}

confirm the result.