Theorem The log logistic distribution has the variate generation property. That is, the inverse cumulative distribution function of a log logistic(λ, κ) random variable can be expressed in closed-form.

Proof The probability density function of a log logistic(λ, κ) random variable is

$$f(x) = \frac{\lambda\kappa(\lambda x)^{\kappa-1}}{(1+(\lambda x)^{\kappa})^2} \qquad x > 0.$$

The cumulative distribution function is

$$F(x) = \frac{(\lambda x)^{\kappa}}{1+(\lambda x)^{\kappa}} \qquad x > 0.$$

Equating the cumulative distribution function to u where $0 < u < 1$ yields the inverse cumulative distribution function

$$F^{-1}(u) = \left(\frac{u}{\lambda^{\kappa}(1-u)}\right)^{1/\kappa} \qquad 0 < u < 1.$$

So a closed form variate generation algorithm for the log logistic distribution is

generate $U \sim U(0,1)$
$X \leftarrow (u/(\lambda^{\kappa}(1-u)))^{1/\kappa}$
return(X)

APPL verification: The APPL statements

```
X := LogLogisticRV(lambda, kappa);
CDF(X);
IDF(X);
```

confirm the result.