Theorem The log logistic distribution has the inverse property. That is, if $X \sim$ log logistic(λ, κ) then $Y = 1/X$ also has the log logistic distribution.

Proof Let the random variable X have the log logistic(λ, κ) distribution with probability density function

$$f(x) = \frac{\lambda\kappa(\lambda x)^{\kappa-1}}{(1+(\lambda x)^{\kappa})^2} \qquad\qquad x > 0.$$

The transformation $Y = g(X) = 1/X$ is a 1–1 transformation from $\mathcal{X} = \{x \,|\, x > 0\}$ to $\mathcal{Y} = \{y \,|\, y > 0\}$ with inverse $X = g^{-1}(Y) = 1/Y$ and Jacobian

$$\frac{dX}{dY} = -\frac{1}{Y^2}.$$

Therefore, by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left|\frac{dx}{dy}\right| \\ &= \frac{\lambda\kappa(\lambda(1/y))^{\kappa-1}}{(1+(\lambda(1/y))^{\kappa})^2} \left|-\frac{1}{y^2}\right| \\ &= \frac{\lambda\kappa\lambda^{\kappa-1}y^{-\kappa-1}}{(1+(\lambda/y)^{\kappa})^2} \\ &= \frac{\lambda^{-\kappa}\kappa y^{\kappa-1}}{(1+(y/\lambda)^{\kappa})^2} \qquad\qquad y > 0, \end{aligned}$$

which is the probability density function of a log logistic$(1/\lambda, \kappa)$ random variable.

APPL verification: The APPL statements

```
assume(c > 0);
X := LogLogisticRV(lambda, kappa);
g := [[x -> 1 / x], [0, infinity]];
Y := Transform(X, g);
```

give the correct probability density function, although it needs to be simplified.