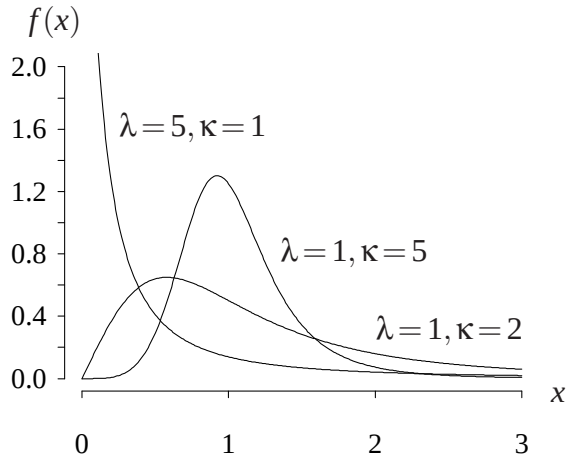
Log-Logistic distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \text{loglogistic}(\lambda, \kappa)$ is used to indicate that the random variable X has the log-logistic distribution with positive scale parameter λ and positive shape parameterκ. A log-logistic random variable X with parameters λ and κ has probability density function

$$f(x) = \frac{\lambda\kappa(\lambda x)^{\kappa-1}}{(1+(\lambda x)^{\kappa})^2} \qquad x > 0$$

for $\lambda > 0$, $\kappa > 0$. The log logistic distribution can be used to model the lifetime of an object, the lifetime of a organism, or a service time. The probability density function with three different parameter settings is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \frac{(\lambda x)^{\kappa}}{1+(\lambda x)^{\kappa}} \qquad x > 0.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = \frac{1}{1+(\lambda x)^{\kappa}} \qquad x > 0.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{\lambda\kappa(\lambda x)^{\kappa-1}}{1+(\lambda x)^{\kappa}} \qquad x > 0.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln(S(x)) = \ln[1+(\lambda x)^{\kappa}] \qquad x > 0.$$

The inverse distribution function of X is

$$F^{-1}(u) = \frac{1}{\lambda}\left(\frac{u}{1-u}\right)^{1/\kappa} \qquad 0 < u < 1.$$

The median of X is

$$\frac{1}{\lambda}.$$

The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = \int_0^\infty \frac{e^{tx}\lambda^\kappa \kappa x^{\kappa-1}}{(1+(\lambda x)^\kappa)^2} dx \qquad t > 0.$$

The characteristic function of X is

$$\phi(t) = E\left[e^{itX}\right] = \int_0^\infty \frac{e^{itx}\lambda^\kappa \kappa x^{\kappa-1}}{(1+(\lambda x)^\kappa)^2} dx \qquad t > 0.$$

The population mean and variance are

$$E[X] = \frac{\pi}{\kappa\lambda\left(\sin\left(\frac{\pi}{\kappa}\right)\right)} \qquad V[X] = \frac{\pi\left(2\kappa\left(1-\cos\left(\frac{\pi}{\kappa}\right)^2\right)+\pi\sin\left(\frac{\pi(\kappa+2)}{\kappa}\right)\right)}{\left(\sin\left(\frac{\pi(\kappa+2)}{\kappa}\right)\right)\left(\left(\cos\left(\frac{\pi}{\kappa}\right)\right)^2-1\right)(\lambda\kappa)^2}$$

APPL verification: The APPL statements

```
X := LogLogisticRV(lambda, kappa);
CDF(X);
SF(X);
HF(X);
CHF(X);
IDF(X);
MGF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the cumulative distribution function, survivor function, hazard function, cumulative hazard function, inverse, moment generating function, population mean, variance, skewness, and kurtosis.