Log-Logistic distribution (from [http://www.math.wm.edu/~leemis/chart/UDR/UDR.html](http://www.math.wm.edu/~leemis/chart/UDR/UDR.html))

The shorthand \( X \sim \text{loglogistic}(\lambda, \kappa) \) is used to indicate that the random variable \( X \) has the log-logistic distribution with positive scale parameter \( \lambda \) and positive shape parameter \( \kappa \). A log-logistic random variable \( X \) with parameters \( \lambda \) and \( \kappa \) has probability density function

\[
f(x) = \frac{\lambda \kappa (\lambda x)^{\kappa - 1}}{(1 + (\lambda x)^\kappa)^2} \quad x > 0
\]

for \( \lambda > 0, \kappa > 0 \). The log logistic distribution can be used to model the lifetime of an object, the lifetime of an organism, or a service time. The probability density function with three different parameter settings is illustrated below.

The cumulative distribution function on the support of \( X \) is

\[
F(x) = P(X \leq x) = \frac{(\lambda x)^\kappa}{1 + (\lambda x)^\kappa} \quad x > 0.
\]

The survivor function on the support of \( X \) is

\[
S(x) = P(X \geq x) = \frac{1}{1 + (\lambda x)^\kappa} \quad x > 0.
\]

The hazard function on the support of \( X \) is

\[
h(x) = \frac{f(x)}{S(x)} = \frac{\lambda \kappa (\lambda x)^{\kappa - 1}}{1 + (\lambda x)^\kappa} \quad x > 0.
\]

The cumulative hazard function on the support of \( X \) is

\[
H(x) = -\ln(S(x)) = \ln[1 + (\lambda x)^\kappa] \quad x > 0.
\]
The inverse distribution function of $X$ is

$$F^{-1}(u) = \frac{1}{\lambda} \left( \frac{u}{1-u} \right)^{1/\kappa} \quad 0 < u < 1.$$ 

The median of $X$ is

$$\frac{1}{\bar{\lambda}}.$$ 

The moment generating function of $X$ is

$$M(t) = E[e^{tX}] = \int_0^\infty \frac{e^{tx} \lambda^\kappa x^{\kappa-1}}{(1 + (\lambda x)^\kappa)^2} dx \quad t > 0.$$ 

The characteristic function of $X$ is

$$\phi(t) = E[e^{itX}] = \int_0^\infty \frac{e^{itx} \lambda^\kappa x^{\kappa-1}}{(1 + (\lambda x)^\kappa)^2} dx \quad t > 0.$$ 

The population mean and variance are

$$E[X] = \frac{\pi}{\kappa \lambda (\sin \left( \frac{\pi}{\kappa} \right))} \quad V[X] = \frac{\pi \left( 2 \kappa \left( 1 - \cos \left( \frac{\pi}{\kappa} \right)^2 \right) + \pi \sin \left( \frac{\pi (\kappa + 2)}{\kappa} \right) \left( \sin \left( \frac{\pi}{\kappa} \right) \right)^2 \right)}{\left( \sin \left( \frac{\pi (\kappa + 2)}{\kappa} \right) \right) \left( \left( \cos \left( \frac{\pi}{\kappa} \right) \right)^2 - 1 \right) (\lambda \kappa)^2}$$

**APPL verification:** The APPL statements

```appl
X := LogLogisticRV(lambda, kappa);
CDF(X);
SF(X);
HF(X);
CHF(X);
IDF(X);
MGF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the cumulative distribution function, survivor function, hazard function, cumulative hazard function, inverse, moment generating function, population mean, variance, skewness, and kurtosis.