Theorem The logistic-exponential distribution has the scaling property. That is, if $X \sim$ logistic-exponential(α, β) then $Y = kX$ also has the logistic-exponential distribution.

Proof Let the random variable X have the logistic-exponential(α, β) distribution with probability density function

$$f(x) = \frac{\alpha\,\beta\,(e^{\alpha\,x} - 1)^{\beta-1}\,e^{\alpha\,x}}{\left(1 + (e^{\alpha\,x} - 1)^{\beta}\right)^2} \qquad\qquad x > 0.$$

Let k be a positive, real constant. The transformation $Y = g(X) = kX$ is a 1–1 transformation from $\mathcal{X} = \{x \,|\, x > 0\}$ to $\mathcal{Y} = \{y \,|\, y > 0\}$ with inverse $X = g^{-1}(Y) = Y/k$ and Jacobian

$$\frac{dX}{dY} = \frac{1}{k}.$$

Therefore, by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y))\left|\frac{dx}{dy}\right| \\ &= \frac{\alpha\,\beta\,\left(e^{\alpha\,y/k} - 1\right)^{\beta-1}\,e^{\alpha\,y/k}}{\left(1 + (e^{\alpha\,y/k} - 1)^{\beta}\right)^2}\left|\frac{1}{k}\right| \\ &= \frac{(\alpha/k)\,\beta\,\left(e^{\alpha\,y/k} - 1\right)^{\beta-1}\,e^{\alpha\,y/k}}{\left(1 + (e^{\alpha\,y/k} - 1)^{\beta}\right)^2} \qquad\qquad y > 0. \end{aligned}$$

which is the probability density function of a logistic-exponential$(\alpha/k, \beta)$ random variable.