**Theorem** The logistic-exponential distribution has the scaling property. That is, if $X \sim \text{logistic-exponential}(\alpha, \beta)$ then $Y = kX$ also has the logistic-exponential distribution.

**Proof** Let the random variable $X$ have the logistic-exponential($\alpha$, $\beta$) distribution with probability density function

$$f(x) = \frac{\alpha \beta (e^{\alpha x} - 1)^{\beta-1} e^{\alpha x}}{(1 + (e^{\alpha x} - 1)^{\beta})^2} \quad x > 0.$$ 

Let $k$ be a positive, real constant. The transformation $Y = g(X) = kX$ is a 1–1 transformation from $\mathcal{X} = \{x \mid x > 0\}$ to $\mathcal{Y} = \{y \mid y > 0\}$ with inverse $X = g^{-1}(Y) = Y/k$ and Jacobian $\frac{dX}{dY} = \frac{1}{k}$.

Therefore, by the transformation technique, the probability density function of $Y$ is

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right|$$

$$= \frac{\alpha \beta (e^{\alpha y/k} - 1)^{\beta-1} e^{\alpha y/k}}{(1 + (e^{\alpha y/k} - 1)^{\beta})^2} \left| \frac{1}{k} \right|$$

$$= \frac{(\alpha/k) \beta (e^{\alpha y/k} - 1)^{\beta-1} e^{\alpha y/k}}{(1 + (e^{\alpha y/k} - 1)^{\beta})^2} \quad y > 0.$$ 

which is the probability density function of a logistic-exponential($\alpha/k$, $\beta$) random variable.