Theorem The exponential distribution is a special case of the logistic exponential distribution when $\beta = 1$.

Proof Let X be a logistic exponential random variable with parameters α and β. The probability density function of X is

$$f_X(x) = \frac{\alpha\beta(e^{\alpha x} - 1)^{\beta-1}e^{\alpha x}}{(1 + (e^{\alpha x} - 1)^{\beta})^2} \qquad x > 0.$$

Setting $\beta = 1$ gives

$$\begin{aligned} f_X(x) &= \frac{\alpha(e^{\alpha x} - 1)^0 e^{\alpha x}}{(1 + (e^{\alpha x} - 1))^2} \\ &= \frac{\alpha e^{\alpha x}}{e^{2\alpha x}} \\ &= \alpha e^{-\alpha x} \qquad x > 0, \end{aligned}$$

which is the probability density function of an exponential random variable with population mean $1/\alpha$.

APPL verification: The APPL statements

```
assume(alpha > 0);
assume(beta > 0);
X := [[x -> alpha * beta * (exp(alpha * x) - 1) ^ (beta - 1) *
            exp(alpha * x) / (1 + (exp(alpha * x) - 1) ^ beta) ^ 2],
      [0, infinity], ["Continuous", "PDF"]];
simplify(subs(beta = 1, X[1][1](x)));
ExponentialRV(alpha);
```

verify the result.