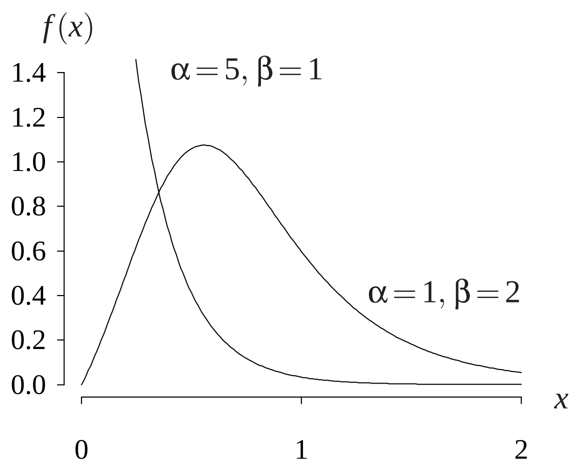
Logistic-exponential distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim$ logistic-exponential(α, β) is used to indicate that the random variable X has the logistic-exponential distribution with positive scale parameter α and positive shape parameter β. A logistic-exponential random variable X with parameters α and β has probability density function

$$f(x) = \frac{\alpha\beta\,(e^{\alpha x}-1)^{\beta-1}\,e^{\alpha x}}{\left(1+(e^{\alpha x}-1)^{\beta}\right)^{2}} \qquad x > 0,$$

for all $\alpha > 0$ and for $\beta > 0$. The probability density function for two different parameter settings is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \frac{(e^{\alpha x}-1)^{\beta}}{1+(e^{\alpha x}-1)^{\beta}} \qquad x > 0.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = \frac{1}{1+(e^{\alpha x}-1)^{\beta}} \qquad x > 0.$$

The hazard function on the support of X is

$$h(x) = \frac{\alpha\beta\,(e^{\alpha x}-1)^{\beta-1}\,e^{\alpha x}}{1+(e^{\alpha x}-1)^{\beta}} \qquad x > 0.$$

The cumulative hazard function on the support of X is mathematically intractable.

$$H(x) = \ln\left(1+(e^{\alpha x}-1)^{\beta}\right) \qquad x > 0.$$

The inverse distribution function of X is

$$F^{-1}(u) = \ln\left(\left(\frac{u}{1-u}\right)^{1/\beta} + 1\right) \Big/ \alpha \qquad\qquad 0 < u < 1.$$

The median of X is

$$\frac{\ln(2)}{\alpha}.$$

The moment generating function and characteristic function of X are mathematically intractable. The population mean, variance, skewness, and kurtosis of X are also mathematically intractable.

APPL verification: The APPL statements

```
X := [[x -> alpha * beta * (exp(alpha * x) - 1) ^ (beta - 1) * exp(alpha * x)/
            (1 + (exp(alpha * x) - 1) ^ beta) ^ 2],[0, infinity],
            ["Continuous", "PDF"]];
CDF(X);
SF(X);
HF(X);
CHF(X);
IDF(X);
```

verify the cumulative distribution function, survivor function, hazard function, cumulative hazard function, and inverse distribution function.