Theorem The logistic distribution has the variate generation property. That is, the inverse cumulative distribution function of a logistic(λ, κ) random variable can be expressed in closed-form.

Proof The probability density function of a logistic(λ, κ) random variable is

$$f(x) = \frac{\lambda^\kappa \kappa e^{(\kappa x)}}{(1 + (\lambda e^x)^\kappa)^2} \qquad -\infty < x < \infty.$$

The cumulative distribution function is

$$F(x) = \frac{\lambda^\kappa e^{(\kappa x)}}{1 + \lambda^\kappa e^x} \qquad -\infty < x < \infty.$$

Equating the cumulative distribution function to u where $0 < u < 1$ yields the inverse cumulative distribution function

$$F^{-1}(u) = -\frac{1}{\kappa} \ln \left(\frac{\lambda^\kappa (1-u)}{u} \right) \qquad 0 < u < 1.$$

So a closed form variate generation algorithm for the logistic distribution is

generate $U \sim U(0, 1)$
$X \leftarrow -\ln \left(\lambda^\kappa (1-u)/u \right) / \kappa$
return(X)

APPL verification: The APPL statements

```
X := LogisticRV(kappa, lambda);
CDF(X);
IDF(X);
```

confirm the result.