**Theorem** The logistic distribution has the scaling property. That is, if $X \sim \text{logistic}(\lambda, \kappa)$ then $Y = cX$ also has the logistic distribution.

**Proof** Let the random variable $X$ have the logistic($\lambda, \kappa$) distribution with probability density function

$$f(x) = \frac{\lambda e^{\kappa x}}{(1 + (\lambda e^{\kappa})^2)} \quad -\infty < x < \infty.$$ 

Let $c$ be a positive, real constant. The transformation $Y = g(X) = cX$ is a 1-1 transformation from $\mathcal{X} = \{x \mid -\infty < x < \infty\}$ to $\mathcal{Y} = \{y \mid -\infty < y < \infty\}$ with inverse $X = g^{-1}(Y) = Y/c$ and Jacobian

$$\frac{dX}{dY} = \frac{1}{c}.$$ 

Therefore, by the transformation technique, the probability density function of $Y$ is

$$f_Y(y) = f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| = \frac{\lambda e^{\kappa y/c}}{(1 + (\lambda e^{y/c})^2)} \left| \frac{1}{c} \right| = \frac{\lambda e^{\kappa y/c}}{(1 + (\lambda e^{y/c})^2)} \quad -\infty < y < \infty.$$

which is the probability density function of a logistic($\lambda c, \kappa/c$) random variable.

**APPL failure:** The APPL statements

```apl
assume(c > 0);
X := LogisticRV(kappa, lambda);
g := [[x -> c * x], [0, infinity]];
Y := Transform(X, g);
```

fail to produce the probability density function of a logistic random variable.