Theorem The logistic distribution has the scaling property. That is, if $X \sim \text{logistic}(\lambda, \kappa)$ then $Y = cX$ also has the logistic distribution.

Proof Let the random variable X have the logistic(λ, κ) distribution with probability density function

$$f(x) = \frac{\lambda^\kappa \kappa e^{(\kappa x)}}{(1 + (\lambda e^x)^\kappa)^2} \qquad -\infty < x < \infty.$$

Let c be a positive, real constant. The transformation $Y = g(X) = cX$ is a 1–1 transformation from $\mathcal{X} = \{x \mid -\infty < x < \infty\}$ to $\mathcal{Y} = \{y \mid -\infty < y < \infty\}$ with inverse $X = g^{-1}(Y) = Y/c$ and Jacobian

$$\frac{dX}{dY} = \frac{1}{c}.$$

Therefore, by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left| \frac{dx}{dy} \right| \\ &= \frac{\lambda^\kappa \kappa e^{(\kappa y/c)}}{(1 + (\lambda e^{y/c})^\kappa)^2} \left| \frac{1}{c} \right| \\ &= \frac{\lambda^\kappa (\kappa/c) e^{(\kappa y/c)}}{(1 + (\lambda^c e^y)^{\kappa/c})^2} \qquad -\infty < y < \infty. \end{aligned}$$

which is the probability density function of a logistic$(\lambda^c, \kappa/c)$ random variable.

APPL failure: The APPL statements

```
assume(c > 0);
X := LogisticRV(kappa, lambda);
g := [[x -> c * x], [0, infinity]];
Y := Transform(X, g);
```

fail to produce the probability density function of a logistic random variable.