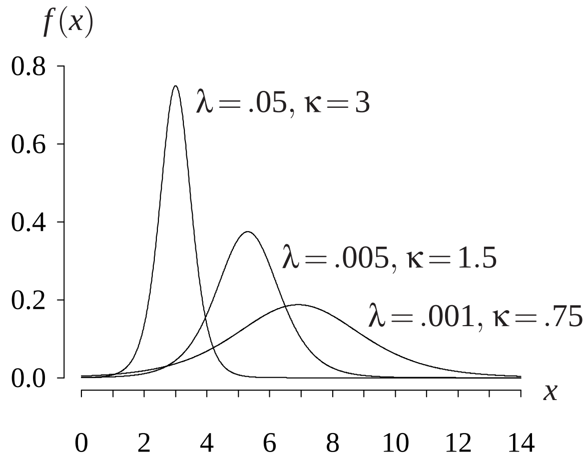
Logistic distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim \text{logistic}(\lambda, \kappa)$ is used to indicate that the random variable X has the logistic distribution with parameters λ and κ. A logistic random variable X with positive scale parameter λ and positive shape parameter κ has probability density function

$$f(x) = \frac{\lambda^\kappa \kappa e^{\kappa x}}{(1 + (\lambda e^x)^\kappa)^2} \qquad -\infty < x < \infty.$$

The probability density function with three different parameter settings is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = \frac{\lambda^\kappa e^{\kappa x}}{1 + \lambda^\kappa e^x} \qquad -\infty < x < \infty.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = \frac{1 + \lambda^\kappa e^x - \lambda^\kappa e^{x\kappa}}{1 + \lambda^\kappa e^x} \qquad -\infty < x < \infty.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \frac{\kappa e^{\kappa x}(\lambda^\kappa + \lambda^{2\kappa} e^x)}{(1 + (\lambda e^x)^\kappa)^2 (1 + \lambda^\kappa e^x - \lambda^\kappa e^{x\kappa})} \qquad -\infty < x < \infty.$$

The inverse distribution function of X is

$$F^{-1}(u) = -\frac{1}{\kappa} \ln\left(\frac{u}{1-u}\right) - \ln\lambda \qquad 0 < u < 1.$$

The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = \int_{-\infty}^{\infty} \frac{\lambda^\kappa \kappa e^{x(t+\kappa)}}{(1 + \lambda^\kappa e^{x\kappa})^2}\, dx.$$

The characteristic function of X is

$$\phi(t) = E\left[e^{itX}\right] = \int_{-\infty}^{\infty} \frac{\lambda^{\kappa}\kappa e^{x(it+\kappa)}}{(1+\lambda^{\kappa}e^{x\kappa})^2}\, dx.$$

The population mean and variance are

$$E[X] = -\ln\lambda \qquad\qquad V[X] = \frac{\pi^2}{3\kappa^2}.$$

The population skewness and kurtosis of X are more complicated expressions.

APPL verification: The APPL statements

```
X := LogisticRV(kappa, lambda);
CDF(X);
SF(X);
HF(X);
IDF(X);
MGF(X);
Mean(X);
Variance(X);
```

verify the cumulative distribution, survivor function, hazard function, inverse distribution function, moment generating function, population mean, and variance.