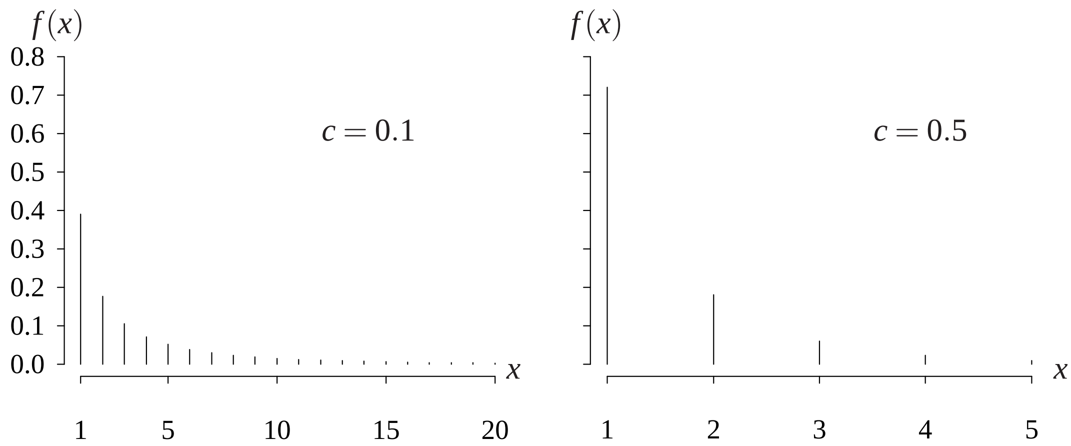
Logarithm distribution (from `http://www.math.wm.edu/~leemis/chart/UDR/UDR.html`)
The shorthand $X \sim \text{logarithm}(c)$ is used to indicate that the random variable X has the logarithm distribution with parameter c. A logarithm random variable X with parameter c has probability mass function

$$f(x) = -\frac{(1-c)^x}{x \ln c} \qquad x = 1, 2, \ldots$$

for any $0 < c < 1$. The probability mass functions for two different values of c are illustrated below.



The cumulative distribution, survivor function, hazard function, cumulative hazard function, and inverse distribution function on the support of X are mathematically intractable.

The moment generating function of X is

$$M(t) = E\left[e^{tX}\right] = \frac{\ln(1 - e^t + e^t c)}{\ln c} \qquad -\infty < t < \infty.$$

The characteristic function of X is

$$\phi(t) = \frac{\ln\left(1 - e^{it} + e^{it} c\right)}{\ln c} \qquad -\infty < t < \infty.$$

The population mean and variance of X are

$$E[X] = \frac{c-1}{c \ln c} \qquad V[X] = \frac{(c-1)(\ln(c) + 1 - c)}{(\ln c)^2 c^2}$$

The skewness and kurtosis can be determined using the APPL code below.

APPL verification: The APPL statements

```
assume(0 < c < 1);
X := [[x -> -(1 - c) ^ x / (x * log(c))],[1 .. infinity], ["Discrete", "PDF"]];
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the population mean, variance, skewness, kurtosis, and moment generating function.