Theorem Random variates from the Laplace distribution with parameters α_1 and α_2 can be generated in closed-form by inversion.

Proof The Laplace(α_1, α_2) distribution has probability density function

$$f(x) = \begin{cases} (1/(\alpha_1 + \alpha_2))e^{x/\alpha_1} & x < 0 \\ (1/(\alpha_1 + \alpha_2))e^{-x/\alpha_2} & x > 0 \end{cases}$$

and cumulative distribution function

$$F(x) = \begin{cases} \frac{\alpha_1}{\alpha_1+\alpha_2}\, e^{x/\alpha_1} & x < 0 \\ 1 - \frac{\alpha_2}{\alpha_1+\alpha_2}\, e^{-x/\alpha_2} & x > 0. \end{cases}$$

Equating the cumulative distribution function to u, where $0 < u < 1$ yields an inverse cumulative distribution function

$$F^{-1}(u) = \begin{cases} \alpha_1 \ln\left(u(1 + \alpha_2/\alpha_1)\right) & 0 < u < \frac{\alpha_1}{\alpha_1+\alpha_2} \\ -\alpha_2 \ln\left((1-u)(1 + \alpha_1/\alpha_2)\right) & \frac{\alpha_1}{\alpha_1+\alpha_2} < u < 1. \end{cases}$$

So a closed-form variate generation algorithm for the exponential(α) distribution is

generate $U \sim U(0, 1)$
if $U < \frac{\alpha_1}{\alpha_1+\alpha_2}$ then
    $X \leftarrow \alpha_1 \ln\left(u(1 + \alpha_2/\alpha_1)\right)$
else
    $X \leftarrow -\alpha_2 \ln\left((1-u)(1 + \alpha_1/\alpha_2)\right)$
return X

APPL failure: The APPL statements

```
assume(alpha1 > 0);
assume(alpha2 > 0);
X := [[x -> (1 / (alpha1 + alpha2)) * exp(x / alpha1),
            (1 / (alpha1 + alpha2)) * exp(-x / alpha2)],
      [-infinity, 0, infinity], ["Continuous", "PDF"]];
CDF(X);
IDF(X);
```

were unable to produce a closed-form equation for the cumulative distribution function or the inverse cumulative distribution function of the Laplace distribution.