**Theorem** If $X \sim \text{Laplace}(\alpha_1, \alpha_2)$, where $\alpha_1 = \alpha_2 = \alpha$, then $Y = |X|$ has the exponential($\alpha$) distribution.

**Proof** Let $X \sim \text{Laplace}(\alpha_1, \alpha_2)$. The cumulative distribution function of $X$ is

$$F_X(x) = \begin{cases} (\alpha_1/(\alpha_1+\alpha_2))e^{x/\alpha_1} & x < 0 \\ 1 - (\alpha_2/(\alpha_1+\alpha_2))e^{-x/\alpha_2} & x \geq 0. \end{cases}$$

When $\alpha_1 = \alpha_2 = \alpha$, the cumulative distribution function of $Y = |X|$ is

$$\begin{aligned} F_Y(y) &= P(Y \leq y) \\ &= P(|X| \leq y) \\ &= P(-y \leq X \leq y) \\ &= F_X(y) - F_X(-y) \\ &= 1 - (1/2)e^{-y/\alpha} - (1/2)e^{-y/\alpha} \\ &= 1 - e^{-y/\alpha} \qquad\qquad y > 0, \end{aligned}$$

which is the cumulative distribution function of an exponential($\alpha$) random variable.

**APPL verification:** The APPL statements

```
assume(alpha > 0);
X := [[x -> exp(x / alpha) / 2, x -> 1 - exp(-x / alpha) / 2],
      [-infinity, 0, infinity], ["Continuous", "CDF"]];
g := [[x -> -x, x -> x], [-infinity, 0, infinity]];
Y := Transform(X, g);
```

yield the probability density function of an exponential($\alpha$) random variable.