**Theorem** If $X \sim \text{Laplace}(\alpha_1, \alpha_2)$, where $\alpha_1 = \alpha_2 = \alpha$, then $Y = |X|$ has the exponential($\alpha$) distribution.

**Proof** Let $X \sim \text{Laplace}(\alpha_1, \alpha_2)$. The cumulative distribution function of $X$ is

$$F_X(x) = \begin{cases} 
\frac{\alpha_1}{\alpha_1 + \alpha_2}e^{x/\alpha_1} & x < 0 \\
1 - \left(\frac{\alpha_2}{\alpha_1 + \alpha_2}\right)e^{-x/\alpha_2} & x \geq 0.
\end{cases}$$

When $\alpha_1 = \alpha_2 = \alpha$, the cumulative distribution function of $Y = |X|$ is

$$F_Y(y) = P(Y \leq y) = P(|X| \leq y) = P(-y \leq X \leq y) = F_X(y) - F_X(-y) = 1 - \frac{1}{2}e^{-y/\alpha} - \frac{1}{2}e^{-y/\alpha} = 1 - e^{-y/\alpha},$$

which is the cumulative distribution function of an exponential($\alpha$) random variable.

**APPL verification:** The APPL statements

```apl
assume(alpha > 0);
X := [[x -> exp(x / alpha) / 2, x -> 1 - exp(-x / alpha) / 2],
    [-infinity, 0, infinity], ["Continuous", "CDF"]];
g := [[x -> -x, x -> x], [-infinity, 0, infinity]]; Y := Transform(X, g);```

yield the probability density function of an exponential($\alpha$) random variable.