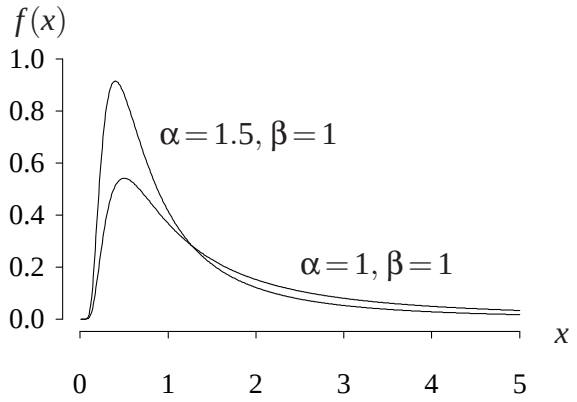
Inverted gamma distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html) The shorthand $X \sim$ inverted gamma(α, β) is used to indicate that the random variable X has the inverted gamma distribution with positive parameters α and β. An inverted gamma random variable X with shape parameter α and scale parameter β has probability density function

$$f(x) = \frac{x^{-(\alpha+1)} e^{-1/(\beta x)}}{\Gamma(\alpha)\beta^{\alpha}} \qquad x > 0.$$

The probability density function with two different parameter settings is illustrated below.



The mode of X is

$$\frac{\beta}{\alpha+1}.$$

The population mean, variance, skewness, and kurtosis of X are

$$E[X] = \frac{1}{\beta(\alpha-1)}, \quad \alpha > 1 \qquad V[X] = \frac{1}{\beta^2(\alpha-1)^2(\alpha-2)}, \quad \alpha > 2$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{4}{(\alpha-1)(\alpha-3)\sqrt{1/(\alpha-2)(\alpha-1)^2}}, \quad \alpha > 3$$

$$E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{3(\alpha-2))(\alpha+5)}{(\alpha-3)(\alpha-4)}, \quad \alpha > 4.$$

APPL verification: The APPL statements

```
X := InvertedGammaRV(alpha, beta);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the population mean, variance, skewness, and kurtosis.