Theorem [UNDER CONSTRUCTION!] If $X_i \sim$ inverse Gaussian(λ_i, μ_i), for $i = 1, 2, \ldots, n$ and $X_1, X_2, \ldots, X_n$ are mutually independent, then $\sum_{i=1}^{n} a_i X_i$ is also inverse Gaussian for nonzero real constants $a_1, a_2, \ldots, a_n$.

Proof [UNDER CONSTRUCTION!] The moment generating function of X_i is

$$M_{X_i}(t) = e^{\lambda_i/\mu_i}\left(1 - \sqrt{1 - \frac{2\mu_i^2 t}{\lambda_i}}\right) \qquad t < \frac{\lambda_i}{2}.$$

The moment generating function of $a_i X_i$ is

$$M_{a_i X_i}(t) = e^{\lambda_i/\mu_i}\left(1 - \sqrt{1 - \frac{2\mu_i^2 a_i t}{\lambda_i}}\right) \qquad t < \frac{\lambda_i}{2}.$$

Since $X_1, X_2, \ldots, X_n$ are mutually independent, the moment generating function of the linear combination is

$$\begin{aligned} M_{a_1X_1+a_2X_2+\cdots+a_nX_n}(t) &= \prod_{i=1}^{n} M_{a_iX_i}(t) \\ &= \prod_{i=1}^{n} e^{\lambda_i/\mu_i}\left(1 - \sqrt{1 - \frac{2\mu_i^2 a_i t}{\lambda_i}}\right) \end{aligned}$$

for $t < \frac{\lambda_i}{2}$.