

Theorem The Rayleigh distribution is a special case of the IDB distribution when $\delta = 2/\alpha$ and $\gamma = 0$.

Proof The IDB distribution has probability density function

$$f(x) = \frac{(1 + \kappa x)\delta x + \gamma}{(1 + \kappa x)^{\gamma/\kappa + 1}} e^{-\delta x^2/2} \quad x \geq 0.$$

When $\delta = 2/\alpha$ and $\gamma = 0$ this reduces to

$$f(x) = \frac{(1 + \kappa x)\frac{2}{\alpha}x + 0}{(1 + \kappa x)^{0+1}} e^{-x^2/\alpha} = \left(\frac{2x}{\alpha}\right) e^{-x^2/\alpha} \quad x \geq 0.$$

which is the probability density function of a Rayleigh distribution.

APPL verification: The APPL statements

```
assume(d >= 0);
assume (gam >= 0);
assume(k >= 0);
assume(alpha > 0);
X := [[x -> ((1 + k * x) * d * x + gam) / ((1 + k * x) ^ (gam / k + 1)) *
        exp((-d * x ^ 2) / 2)], [0, infinity], ["Continuous", "PDF"]];
g := subs([d = 2 / alpha, gam = 0], X[1][1](x));
Y := [[x -> 2 * x / alpha * exp((-x ^ 2) / alpha)],
      [0, infinity], ["Continuous", "PDF"]];
h := Y[1][1](x);
```

yield identical forms of the probability density function, so the Rayleigh is a special case of the IDB distribution.