Theorem [UNDER CONSTRUCTION] The Erlang distribution is a special case of the hypoexponential distribution when $\alpha = \vec{\alpha}$.

Proof [UNDER CONSTRUCTION] Let the random variable X have the hypoexponential distribution with probability density function

$$f(x) = \sum_{i=1}^{n} (1/\alpha_i) e^{-x/\alpha_i} \left(\prod_{j=1, j \neq i}^{n} \frac{\alpha_i}{\alpha_i - \alpha_j} \right) \qquad x > 0.$$

Setting $\alpha_i = \alpha, i = 1, 2, \ldots, n$ yields the probability density function

$$f(x) = \frac{x^{n-1} e^{-x/\alpha}}{\alpha^n (n-1)!} \qquad x > 0.$$

which is the probability density function of an Erlang(α, n) random variable.