Theorem The hypoexponential distribution has the convolution property. That is, if $X_i \sim$ hypoexponential$(\vec{\alpha_i})$, $i = 1, 2, \ldots, n$, are independent random variables then $Y = \sum_{i=1}^{n} X_i$ also has the hypoexponential distribution.

Proof Let the random variable X_1 have the hypoexponential$(\vec{\alpha_1})$ distribution. Let the random variable X_2 have the hypoexponential$(\vec{\alpha_2})$ distribution. Assume X_1 and X_2 are independent. Then,

$$X_1 = T_1 + T_2 + \cdots + T_m,$$

where $T_j \sim$ exponential(α_j), $j = 1, 2, \ldots, m$. Furthermore,

$$X_2 = S_1 + S_2 + \cdots + S_r,$$

where $S_k \sim$ exponential(α_k), $k = 1, 2, \ldots, r$. Let $Y = X_1 + X_2$. Then,

$$Y = \sum_{j=1}^{m} T_j + \sum_{k=1}^{r} S_k,$$

which has the hypoexponential distribution. An inductive argument can be used to prove the result for $n > 2$.