**Hypergeometric distribution** (from [http://www.math.wm.edu/~leemis/chart/UDR/UDR.html](http://www.math.wm.edu/~leemis/chart/UDR/UDR.html))

The shorthand $X \sim \text{hypergeometric}(n_1, n_2, n_3)$ is used to indicate that the random variable $X$ has the hypergeometric distribution for some nonnegative integer parameters $n_1, n_2,$ and $n_3$, where $n_1, n_2 \in \{0, 1, 2, \ldots, n_3\}$. A hypergeometric random variable $X$ for parameters $n_1, n_2,$ and $n_3$ has probability mass function

$$f(x) = \binom{n_1}{x} \binom{n_3-n_1}{n_2-x} \binom{n_3}{n_2}.$$

for $x = \max\{0, n_1 + n_2 - n_3\}, \ldots, \min\{n_1, n_2\}$. The hypergeometric distribution is used for sampling without replacement from a finite population of items. More specifically, a hypergeometric random variable $X$ is the number of defective items in a sample of size $n_2$ items drawn at random and without replacement from a lot of $n_3$ items which contains $n_1$ defective items. Applications include acceptance sampling from quality control and animal population size estimation using tagging with capture/recapture. The probability mass function for $n_1 = 15$, $n_2 = 10$, and $n_3 = 50$ is illustrated below.

The cumulative distribution function on the support of $X$ is

$$F(x) = P(X \leq x) = \sum_{k=0}^{x} \binom{n_1}{k} \binom{n_3-n_1}{n_2-k} \binom{n_3}{n_2} \quad x = \max\{0, n_1 + n_2 - n_3\}, \ldots, \min\{n_1, n_2\}.$$

The moment generating function (from Wikipedia) of $X$ is

$$M_X(t) = \binom{n_3-n_1}{n_2} \binom{n_3}{n_2} 2F_1 \left( -n_2, -n_1, n_3 - n_1 - n_2 + 1, e^t \right),$$

where $2F_1$ is the hypergeometric function defined by

$$2F_1(a, b, c, z) = 1 + \frac{ab}{c} \cdot \frac{z}{1!} + \frac{a(a+1)b(b+1)}{c(c+1)} \cdot \frac{z^2}{2!} + \frac{a(a+1)(a+2)b(b+1)(b+2)}{c(c+1)(c+2)} \cdot \frac{z^3}{3!} + \cdots.$$
The population mean, variance, and skewness of $X$ are

\[
E[X] = \frac{n_2n_1}{n_3} \quad V[X] = \frac{n_2n_3(1 - n_1)(n_3 - n_2)}{n_3^2(n_3 - 1)}
\]

\[
E \left[ \left( \frac{X - \mu}{\sigma} \right)^3 \right] = \frac{(n_3 - 2n_1)(n_3 - 1)^{1/2}(n_3 - 2n_2)}{[n_2n_1(n_3 - n_1)(n_3 - n_2)]^{1/2}(n_3 - 2)}.
\]
**APPL verification:** The APPL statements

\[
X := \text{HypergeometricRV}(n3, n1, n2);
\]
\[
\text{Mean}(X);
\]
\[
\text{Variance}(X);
\]
\[
\text{Skewness}(X);
\]

return complicated expressions for the population mean, variance, and skewness.