Theorem A hyperexponential random variable with a vector of parameters

$$(\alpha_1,\, \alpha_2,\, \ldots,\, \alpha_n) = (\alpha,\, \alpha,\, \ldots,\, \alpha)$$

follows an exponential distribution with parameter α.

Proof Let X be a hyperexponential random variable with parameter $\vec{\alpha}$ such that $\alpha_i = \alpha$ for all $i = 1, 2, \ldots, n$ ($|\vec{\alpha}| = n$). We also restrict p_i such that $p_i > 0$ and $\sum_{i=1}^{n} p_i = 1$. Then by definition, X has probability density function

$$\begin{aligned} f_X(x) &= \sum_{i=1}^{n} \frac{p_i}{\alpha_i}\, e^{-x/\alpha_i} \\ &= \sum_{i=1}^{n} \frac{p_i}{\alpha}\, e^{-x/\alpha} \\ &= \frac{e^{-x/\alpha}}{\alpha} \sum_{i=1}^{n} p_i \\ &= \frac{e^{-x/\alpha}}{\alpha} \cdot 1 \\ &= \frac{1}{\alpha} e^{-x/\alpha} \qquad\qquad x > 0, \end{aligned}$$

which is the probability density function of an exponential random variable with population mean α.

APPL illustration: The APPL statement

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HyperExponentialRV([1 / 2, 1 / 3, 1 / 6],[a, a, a]);
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demonstrates the result for a test case of $n = 3$ and $p_1 = 1/2$, $p_2 = 1/3$, $p_3 = 1/6$. APPL would not take general parameters for the p_i, nor would it take the general length n for the parameter list sizes.

Note that APPL uses $1/\alpha$ as opposed to α in the statement above.