Hyperexponential distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)

The shorthand \( X \sim \text{hyperexponential}(\vec{\alpha}, \vec{p}) \) is used to indicate that the random variable \( X \) has the hyperexponential distribution with parameters \( \vec{\alpha} \) and \( \vec{p} \). A hyperexponential random variable \( X \) with parameters \( \vec{\alpha} \) and \( \vec{p} \) has probability density function

\[
f(x) = \sum_{i=1}^{n} \frac{p_i}{\alpha_i} e^{-x/\alpha_i}, \quad x > 0
\]

for all \( \alpha_i, p_i > 0 \) such that \( \sum_{i=1}^{n} p_i = 1 \). The probability density function for \( \vec{\alpha} = (0.25, 0.5, 1) \) and \( \vec{p} = (0.5, 0.25, 0.25) \) is illustrated below.

![Hyperexponential distribution graph](image)

The cumulative distribution function on the support of \( X \) is

\[
F(x) = P(X \leq x) = 1 - \sum_{i=1}^{n} p_i e^{-x/\alpha_i}, \quad x > 0.
\]

The survivor function on the support of \( X \) is

\[
S(x) = P(X \leq x) = \sum_{i=1}^{n} p_i e^{-x/\alpha_i}, \quad x > 0.
\]

The hazard function on the support of \( X \) is

\[
h(x) = \frac{f(x)}{S((x))} = \frac{\sum_{i=1}^{n} \frac{p_i}{\alpha_i} e^{-x/\alpha_i}}{\sum_{i=1}^{n} p_i e^{-x/\alpha_i}}, \quad x > 0.
\]

The inverse distribution function and characteristic function are both mathematically intractable.

The moment generating function over the support of \( X \) is

\[
M(t) = E[e^{tX}] = \sum_{i=1}^{n} \frac{p_i}{1 - \alpha_i t}, \quad |t| < \frac{1}{\min_j \alpha_j}.
\]
The population mean of $X$ is

$$E[X] = \sum_{i=1}^{n} p_i \alpha_i.$$ 

The population skewness and kurtosis of $X$ are all mathematically intractable.

**APPL verification:** The APPL statements

```appl
X := HyperExponentialRV([0.1, 0.3, 0.6], [alpha1, alpha2, alpha3]);
CDF(X);
SF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the population mean, variance, skewness, kurtosis, and moment generating function for the special case of $n = 3$ and $\vec{p} = (0.1, 0.3, 0.6)$. 