**Theorem** Random variates from the hyperbolic-secant distribution can be generated in closed-form by inversion.

**Proof** The hyperbolic-secant distribution has probability density function
\[ f(x) = \text{sech} (\pi x) \quad -\infty < x < \infty, \]
where the hyperbolic-secant function is defined by
\[ \text{sech}(z) = \frac{2}{e^z + e^{-z}} \]
for \(-\infty < z < \infty\). The cumulative distribution function is
\[ F(x) = \int_{-\infty}^{x} \frac{2}{e^z + e^{-z}} \, dz = \frac{2}{\pi} \arctan (e^{\pi x}) \quad -\infty < x < \infty. \]
Equating the cumulative distribution function to \(u\), where \(0 < u < 1\), yields an inverse cumulative distribution function
\[ F^{-1}(u) = \frac{1}{\pi} \ln \left[ \tan \left( \frac{\pi u}{2} \right) \right] \quad 0 < u < 1. \]
So a closed-form variate generation algorithm using inversion for the hyperbolic-secant distribution is
\[
\begin{align*}
\text{generate } U & \sim \text{U}(0, 1) \\
X & \leftarrow \frac{1}{\pi} \ln \left[ \tan \left( \frac{\pi u}{2} \right) \right] \\
& \text{return}(X)
\end{align*}
\]
**APPL verification:** The APPL statements
\[
\begin{align*}
X & := \text{HyperbolicSecantRV}(); \\
\text{CDF}(X); \\
\text{IDF}(X); \\
\end{align*}
\]
produce the correct inverse distribution function.