Theorem Random variates from the hyperbolic-secant distribution can be generated in closed-form by inversion.

Proof The hyperbolic-secant distribution has probability density function

$$f(x) = \operatorname{sech}(\pi x) \qquad\qquad -\infty < x < \infty,$$

where the hyperbolic-secant function is defined by

$$\operatorname{sech}(z) = \frac{2}{e^z + e^{-z}}$$

for $-\infty < z < \infty$. The cumulative distribution function is

$$F(x) = \int_{-\infty}^{x} \frac{2}{e^{\pi z} + e^{-\pi z}}\, dz = \frac{2}{\pi} \arctan\left(e^{\pi x}\right) \qquad\qquad -\infty < x < \infty.$$

Equating the cumulative distribution function to u, where $0 < u < 1$, yields an inverse cumulative distribution function

$$F^{-1}(u) = \frac{1}{\pi} \ln\left[\tan\left(\frac{\pi u}{2}\right)\right] \qquad\qquad 0 < u < 1.$$

So a closed-form variate generation algorithm using inversion for the hyperbolic-secant distribution is

generate $U \sim U(0, 1)$
$X \leftarrow \frac{1}{\pi} \ln\left[\tan\left(\frac{\pi u}{2}\right)\right]$
return(X)

APPL verification: The APPL statements

```
X := HyperbolicSecantRV();
CDF(X);
IDF(X);
```

produce the correct inverse distribution function.