Theorem Random variates from the Gompertz distribution with parameters δ and κ can be generated in closed-form by inversion.

Proof The Gompertz(δ, κ) distribution has cumulative distribution function

$$F(x) = 1 - e^{-\delta\,(\kappa^x - 1)/\ln(\kappa)} \qquad\qquad x > 0.$$

Equating the cumulative distribution function to u, where $0 < u < 1$ yields an inverse cumulative distribution function

$$F^{-1}(u) = \frac{\ln(1 - \ln(1-u)\ln(\kappa)/\delta)}{\ln(\kappa)} \qquad\qquad 0 < u < 1.$$

So a closed-form variate generation algorithm using inversion for the Gompertz(δ, κ) distribution is

generate $U \sim U(0, 1)$
$X \leftarrow \ln(1 - \ln(1-U)\ln(\kappa)/\delta)/\ln(\kappa)$
return(X)

APPL verification: The APPL statements

```
X := GompertzRV(delta, kappa);
CDF(X);
IDF(X);
```

verify the inverse distribution function of a Gompertz random variable.