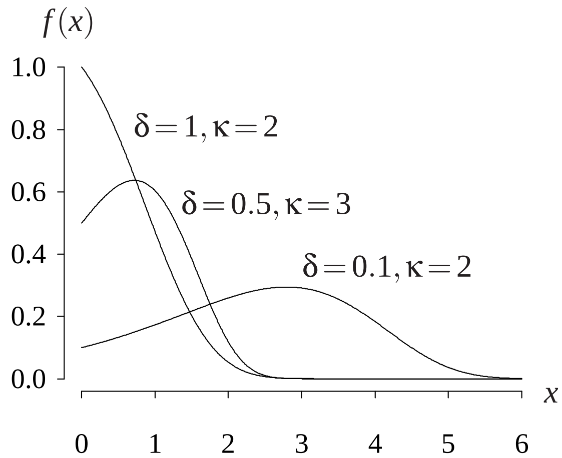
Gompertz distribution (from `http://www.math.wm.edu/~leemis/chart/UDR/UDR.html`)
The shorthand $X \sim \text{Gompertz}(\delta, \kappa)$ is used to indicate that the random variable X has the Gompertz distribution with parameters δ and κ. A Gompertz random variable X with shape parameters δ and κ has probability density function

$$f(x) = \delta \kappa^x e^{-\delta(\kappa^x - 1)/\ln(\kappa)} \qquad x > 0,$$

for all $\delta > 0$ and $\kappa > 1$. The Gompertz distribution is used to model adult lifetimes by actuaries. The probability density function for three parameter combinations is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = 1 - e^{-\delta(\kappa^x - 1)/\ln(\kappa)} \qquad x > 0.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = e^{-\delta(\kappa^x - 1)/\ln(\kappa)} \qquad x > 0.$$

The hazard function on the support of X is

$$h(x) = \delta \kappa^x \qquad x > 0.$$

The cumulative hazard function on the support of X is

$$H(x) = \frac{\delta(\kappa^x - 1)}{\ln(\kappa)} \qquad x > 0.$$

The inverse distribution function of X is

$$F^{-1}(u) = -\frac{\ln(\delta) - \ln(\delta - \ln(1-u)\ln(\kappa))}{\ln(\kappa)} \qquad 0 < u < 1.$$

The median of X is

$$-\frac{\ln(\delta)-\ln(\delta+\ln(2)\ln(\kappa))}{\ln(\kappa)}.$$

The moment generating function and the characteristic function of X cannot be expressed in closed form. The population mean, variance, skewness, and kurtosis of X are mathematically intractable.

APPL verification: The APPL statements

```
assume(delta>0);
assume(kappa>1);
X := [[x -> delta * kappa ^ x * exp(-delta * (kappa ^ x-1) / ln(kappa))],
        [0,infinity],["Continuous", "PDF"]];
CDF(X);
SF(X);
HF(X);
CHF(X);
IDF(X);
```

verify the cumulative distribution function, survivor function, hazard function, cumulative hazard function, and inverse distribution function.