Theorem If $X \sim \text{exponential}(\lambda)$, then

$$\lfloor X \rfloor \sim \text{geometric}\left(1 - e^{-\lambda}\right)$$

Proof Let $Y = \lfloor X \rfloor$. The probability density function of X is

$$f_X(x) = \lambda e^{-\lambda x} \qquad\qquad x > 0,$$

for $\lambda > 0$. The probability mass function of Y is

$$\begin{aligned}
f_Y(y) &= P(Y = y) \\
&= P\left(\lfloor X \rfloor = y\right) \\
&= P\left(y \le X < y + 1\right) \\
&= \int_y^{y+1} f_X(x)dx \\
&= \int_y^{y+1} \lambda e^{-\lambda x} dx \\
&= \left[-e^{-\lambda x}\right]_y^{y+1} \\
&= e^{-\lambda y} - e^{-\lambda(y+1)} \\
&= \left(1 - e^{-\lambda}\right)\left(e^{-\lambda}\right)^y \qquad\qquad y = 0, 1, 2, \ldots,
\end{aligned}$$

which is the probability mass function of a geometric$(1 - e^{-\lambda})$ random variable. So a closed-form variate generation algorithm using inversion for the geometric(p) distribution is

$\lambda \leftarrow -\ln(1 - p)$
generate $U \sim U(0, 1)$
$X \leftarrow -\frac{1}{\lambda}\ln(1 - U)$
$Y \leftarrow \lfloor X \rfloor$
return(Y)