Theorem The minimum of n mutually independent and identically distributed geometric random variables with parameter $0 < p < 1$ is geometric.

Proof Let $X_1,\ X_2,\ \ldots,\ X_n$ be n mutually independent and identically distributed geometric random variables with parameter p, where $0 < p < 1$. The goal is to find the probability distribution of $Y = \min\{X_1,\ X_2,\ \ldots,\ X_n\}$. The probability mass function of X_i is

$$f_{X_i}(x) = p(1-p)^x \qquad\qquad x = 0,\ 1,\ 2, \ldots$$

for $i = 1,\ 2,\ \ldots,\ n$. The associated survivor function of X_i is

$$\begin{aligned} P(X_i \ge x) &= \sum_{i=x}^{\infty} p(1-p)^i \\ &= p(1-p)^x \sum_{i=0}^{\infty} (1-p)^i \\ &= p(1-p)^x \frac{1}{p} \\ &= (1-p)^x \qquad\qquad x = 0,\ 1,\ 2,\ \ldots \end{aligned}$$

for $i = 1,\ 2,\ \ldots,\ n$. It follows that the survivor function of Y is

$$\begin{aligned} S_Y(y) &= P(Y \ge y) \\ &= P(X_1 \ge y) P(X_2 \ge y) \ldots P(X_n \ge y) \\ &= (1-p)^y (1-p)^y \ldots (1-p)^y \\ &= (1-p)^{ny} \qquad\qquad y = 0,\ 1,\ 2,\ \ldots, \end{aligned}$$

which is the survivor function of a geometric random variable. So, it follows that the minimum of n mutually independent and identically distributed geometric random variables has the geometric distribution.

APPL verification: The APPL statements

```
X := GeometricRV(p);
MinimumIID(X, n);
Y := GeometricRV(1 - (1 - p) ^ n);
```

confirm the result by returning the same probability mass function for `X` and `Y`.