Theorem A Pareto random variable is equivalent to the sum of the constant δ and a generalized Pareto random variable with $\gamma = 0$.

Proof Let X be a generalized Pareto random variable with probability density function

$$f(x) = \left(\gamma + \frac{\kappa}{x+\delta}\right)\left(1 + \frac{x}{\delta}\right)^{-\kappa} e^{-\gamma x} \qquad x > 0.$$

When $\gamma = 0$, this reduces to

$$f(x) = \left(\frac{\kappa}{x+\delta}\right)\left(1 + \frac{x}{\delta}\right)^{-\kappa} \qquad x > 0.$$

The transformation $Y = g(X) = X + \delta$ is a 1–1 transformation from $\mathcal{X} = \{x \,|\, x > 0\}$ to $\mathcal{Y} = \{y \,|\, y > \delta\}$ with inverse $X = g^{-1}(Y) = Y - \delta$ and associated Jacobian

$$\frac{dX}{dY} = 1.$$

Using the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y)) \left|\frac{dx}{dy}\right| \\ &= \left(\frac{\kappa}{y-\delta+\delta}\right)\left(1 + \frac{y-\delta}{\delta}\right)^{-\kappa} |1| \\ &= \frac{\kappa\delta^{\kappa}}{y^{\kappa+1}} \qquad y > \delta, \end{aligned}$$

which is the probability density function of a Pareto random variable.

APPL verification: The APPL statements

```
X := GeneralizedParetoRV(myGamma, delta, kappa);
Y := [[y -> simplify(subs(myGamma = 0, X[1][1](y)))],
      [0, infinity], ["Continuous", "PDF"]];
g := [[x -> x + delta], [0,infinity]];
Z := Transform(Y, g);
```

have problems in the `Transform` function call. A work-around is given below.

```
X := GeneralizedParetoRV(myGamma, delta, kappa);
simplify(subs(myGamma = 0, X[1][1](y - delta)));
```