Theorem [UNDER CONSTRUCTION!] The limiting distribution of the generalized gamma(α, β, γ) distribution is the log normal(μ, σ) distribution, as $\beta \to \infty$, $\alpha \to \infty$, $\gamma \to 0$, $\gamma^2\beta \to 1/\sigma^2$, and $\alpha\beta^{(1/\gamma)} \to \mu$.

Proof [UNDER CONSTRUCTION!] Let the random variable X have the generalized gamma(α, β, γ) distribution with probability density function

$$f(x) = \frac{\gamma\, x^{\gamma\,\beta-1} e^{-(x/\alpha)^{\gamma}}}{\alpha^{\gamma\,\beta}\Gamma\,(\beta)} \qquad\qquad x > 0.$$

Taking the limit as $\beta \to \infty$ yields

$$\lim_{\beta\to\infty} \frac{\gamma\, x^{\gamma\,\beta-1} e^{-(x/\alpha)^{\gamma}}}{\alpha^{\gamma\,\beta}\Gamma\,(\beta)} =$$

The result appears on page 113 of Forbes, Evans, Hastings, and Peacock (2011), *Statistical Distributions*, Fourth Edition, John Wiley and Sons.