

Theorem The gamma distribution is a special case of the generalized gamma distribution when $\gamma = 1$.

Proof A generalized gamma random variable X has probability density function

$$f(x) = \frac{\gamma}{\alpha^{\gamma}\Gamma(\beta)} x^{\gamma\beta-1} e^{-(x/\alpha)^{\gamma}} \quad x > 0.$$

When $\gamma = 1$, this reduces to

$$f(x) = \frac{1}{\alpha^{\beta}\Gamma(\beta)} x^{\beta-1} e^{-x/\alpha} \quad x > 0,$$

which is the probability density function of the gamma distribution.

Maple verification: The Maple statements

```
X := [[x -> (myGamma / (alpha ^ (myGamma * beta) * GAMMA(beta))) *  
          x ^ (myGamma * beta - 1) * exp(-(x / alpha) ^ myGamma)],  
      [0, infinity], ["Continuous", "PDF"]];  
simplify(subs(myGamma = 1, X[1][1](x)));
```

yield the probability density function of the gamma distribution as parameterized in the proof.