Theorem The Erlang distribution is a special case of the gamma distribution when $\beta = n$.

Proof The gamma distribution has probability density function

$$f(x) = \frac{1}{\alpha^{\beta}\Gamma(\beta)} x^{\beta-1} e^{-x/\alpha} \qquad x > 0.$$

When $\beta = n$, where n is a positive integer, this becomes

$$f(x) = \frac{1}{\alpha^{n}\Gamma(n)} x^{n-1} e^{-x/\alpha} \qquad x > 0,$$

or

$$f(x) = \frac{1}{\alpha^{n}(n-1)!} x^{n-1} e^{-x/\alpha} \qquad x > 0,$$

which is the probability density function of an Erlang random variable with shape parameter n and scale parameter α.

APPL verification: The APPL statements

```
assume(alpha > 0);
assume(beta > 0);
X := [[x -> (1 / (alpha ^ beta * GAMMA(beta))) * x ^ (beta - 1) * exp(-x / alpha)],
      [0, infinity], ["Continuous", "PDF"]];
subs(beta = n, X[1][1](x));
```

confirm the result.