Theorem The chi-square distribution is a special case of the gamma distribution when $n = 2\beta$ and $\alpha = 2$.

Proof The gamma distribution has probability density function

$$f(x) = \frac{1}{\alpha^\beta \Gamma(\beta)} x^{\beta-1} e^{-x/\alpha} \qquad x > 0.$$

When $n = 2\beta$ and $\alpha = 2$, this reduces to

$$f(x) = \frac{1}{2^{n/2} \Gamma(n/2)} x^{n/2-1} e^{-x/2} \qquad x > 0.$$

which is the probability density function of a chi-square random variable with n degrees of freedom.

APPL verification: The APPL statements

```
assume(alpha > 0);
assume(beta > 0);
X := [[x -> (1 / (alpha ^ beta * GAMMA(beta))) * x ^ (beta - 1) * exp(-x / alpha)],
      [0, infinity], ["Continuous", "PDF"]];
subs({alpha = 2, beta = n / 2}, X[1][1](x));
ChiSquareRV(n);
```

yield identical probability density functions.