Theorem If $X \sim F(n_1, n_2)$, the limiting distribution of $n_1 X$ as $n_2 \to \infty$ is the chi-square distribution with n_1 degrees of freedom.

Proof (contributed on December 25, 2014 by Professor Rubén Becerril Borja at Universidad Autónoma Metropolitana—Iztapalapa) Let the random variable X have the $F(n_1, n_2)$ distribution. Therefore

$$X = \frac{U/n_1}{V/n_2}$$

where $U \sim \chi^2(n_1)$ and $V \sim \chi^2(n_2)$ are independent random variables, or

$$n_1 X = \frac{U}{V/n_2}.$$

Since V is a chi-squared variable with n_2 degrees of freedom, it can be written as the sum of n_2 independent and identically distributed chi-squared variables with 1 degree of freedom, so in the denominator of the previous expression can be written as

$$\frac{Y_1 + Y_2 + \cdots + Y_{n_2}}{n_2}$$

with $Y_1, Y_2, \ldots, Y_{n_2}$ mutually independent $\chi^2(1)$ random variables. By the Strong Law of Large Numbers,

$$\frac{V}{n_2} = \frac{Y_1 + Y_2 + \cdots + Y_{n_2}}{n_2} \xrightarrow{\text{a.s.}} E(Y_1) \qquad \text{as } n_2 \to \infty$$

and $E(Y_1) = 1$, which means

$$\lim_{n_2 \to \infty} n_1 X = \lim_{n_2 \to \infty} \frac{U}{V/n_2} \stackrel{\text{a.s.}}{=} U,$$

proving the desired result.