**Theorem** If $X \sim F(n_1, n_2)$, the limiting distribution of $n_1X$ as $n_2 \to \infty$ is the chi-square distribution with $n_1$ degrees of freedom.

**Proof** (contributed on December 25, 2014 by Professor Rubén Becerril Borja at Universidad Autónoma Metropolitana—Iztapalapa) Let the random variable $X$ have the $F(n_1, n_2)$ distribution. Therefore

$$X = \frac{U}{n_1} \frac{V}{n_2}$$

where $U \sim \chi^2(n_1)$ and $V \sim \chi^2(n_2)$ are independent random variables, or

$$n_1X = \frac{U}{V/n_2}.$$ 

Since $V$ is a chi-squared variable with $n_2$ degrees of freedom, it can be written as the sum of $n_2$ independent and identically distributed chi-squared variables with 1 degree of freedom, so in the denominator of the previous expression can be written as

$$\frac{Y_1 + Y_2 + \cdots + Y_{n_2}}{n_2}$$

with $Y_1, Y_2, \ldots, Y_{n_2}$ mutually independent $\chi^2(1)$ random variables. By the Strong Law of Large Numbers,

$$\frac{V}{n_2} = \frac{Y_1 + Y_2 + \cdots + Y_{n_2}}{n_2} \xrightarrow{a.s.} E(Y_1) \quad \text{as } n_2 \to \infty$$

and $E(Y_1) = 1$, which means

$$\lim_{n_2 \to \infty} n_1X = \lim_{n_2 \to \infty} \frac{U}{V/n_2} \xrightarrow{a.s.} U,$$

proving the desired result.