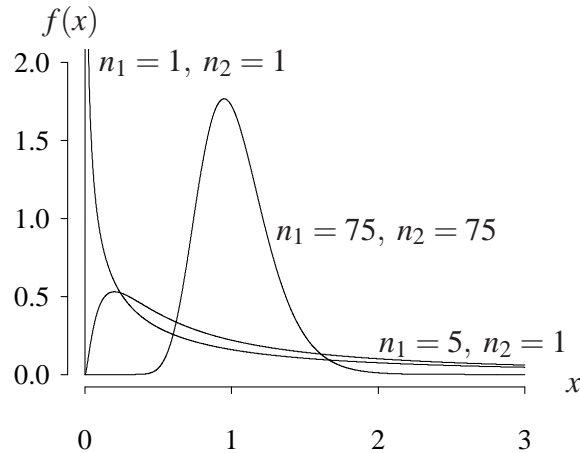


**F distribution** (from <http://www.math.wm.edu/~leemis/chart/UDR/UDR.html>)

The shorthand  $X \sim F(n_1, n_2)$  is used to indicate that the random variable  $X$  has the  $F$  distribution with parameters  $n_1$  and  $n_2$ , which are positive integers known as the degrees of freedom for the numerator and the degrees of freedom for the denominator. The  $F$  distribution is also known as the *variance ratio distribution* and the *Fisher–Snedecor distribution*. An  $F$  random variable  $X$  with  $n_1$  and  $n_2$  degrees of freedom has probability density function

$$f(x) = \frac{\Gamma((n_1 + n_2)/2)(n_1/n_2)^{n_1/2} x^{n_1/2 - 1}}{\Gamma(n_1/2)\Gamma(n_2/2)[(n_1/n_2)x + 1]^{(n_1 + n_2)/2}} \quad x > 0,$$

for  $n_1 = 1, 2, \dots$  and  $n_2 = 1, 2, \dots$ . The  $F$  distribution is used for statistical inference concerning ratios of variances of two normal populations. The  $F$  distribution is used for statistical inference concerning ratios of rates of two exponential populations. The probability density function is plotted below for three parameter combinations:  $n_1 = 1$  and  $n_2 = 1$ ;  $n_1 = 5$  and  $n_2 = 1$ ;  $n_1 = 75$  and  $n_2 = 75$ .



The cumulative distribution function, survivor function, hazard function, cumulative hazard function, inverse distribution function, median, moment generating function, and characteristic function can't be written as closed-form expressions. The mode of  $X$  is

$$\frac{n_2(n_1 - 2)}{n_1(n_2 + 2)} \quad n_1 > 2.$$

The population mean, variance, skewness, and kurtosis of  $X$  are

$$E[X] = \frac{n_2}{n_2 - 2} \quad n_2 > 2$$

$$V[X] = \frac{2n_2^2(n_1 + n_2 - 2)}{n_1(n_2 - 2)^2(n_2 - 4)} \quad n_2 > 4$$

$$E \left[ \left( \frac{X - \mu}{\sigma} \right)^3 \right] = \frac{(2n_1 + n_2 - 2)\sqrt{8(n_2 - 4)}}{(n_2 - 6)\sqrt{n_1(n_1 + n_2 - 2)}} \quad n_2 > 6$$

$$E \left[ \left( \frac{X - \mu}{\sigma} \right)^4 \right] = \frac{3(n_2 - 4)(10n_1^2 + n_1^2 n_2 - 20n_1 + 8n_1 n_2 + n_1 n_2^2 + 16 - 16n_2 + 4n_2^2)}{n_1(n_1 + n_2 - 2)(n_2 - 6)(n_2 - 8)} \quad n_2 > 8.$$

**APPL verification:** The APPL statements

```
X := FRV(n1, n2);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
```

verify the population mean, variance, skewness, and kurtosis.