Theorem Random variates from the extreme value (Gumbel) distribution with parameters α and β can be generated in closed-form by inversion.

Proof The extreme value(α, β) distribution has probability density function

$$f(x) = (\beta/\alpha)\, e^{x\beta - e^{x\beta}/\alpha} \qquad\qquad -\infty < x < \infty$$

and cumulative distribution function

$$F(x) = \left(e^{(e^{x\beta}/\alpha)} - 1\right) e^{-e^{x\beta}/\alpha} \qquad\qquad -\infty < x < \infty.$$

Equating the cumulative distribution function to u, where $0 < u < 1$ yields an inverse cumulative distribution function

$$F^{-1}(u) = \frac{\ln(\alpha) + \ln\left(\ln\left(-\left(u-1\right)^{-1}\right)\right)}{\beta} \qquad\qquad 0 < u < 1.$$

So a closed-form variate generation algorithm using inversion for the extreme value(α, β) distribution is

generate $U \sim U(0, 1)$
$X \leftarrow \left[\ln(\alpha) + \ln\left(\ln\left(-\left(u-1\right)^{-1}\right)\right)\right]/\beta$
return(X)

APPL verification: The APPL statements

```
X := ExtremeValueRV(alpha, beta);
CDF(X);
IDF(X);
```

verify the inverse distribution function of an extreme value random variable.