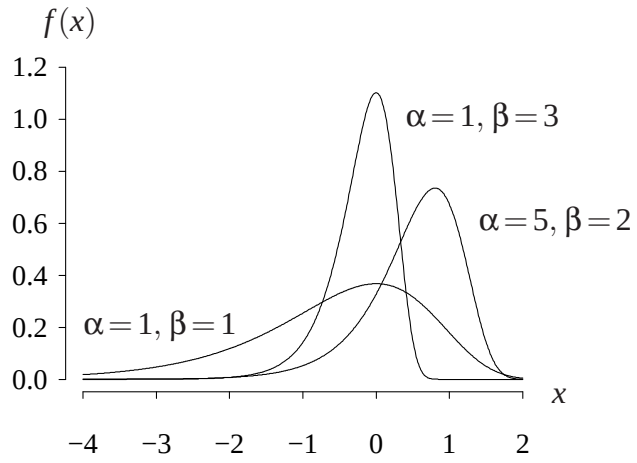
Extreme value distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html) The shorthand $X \sim$ extreme value(α, β) is used to indicate that the random variable X has the extreme value distribution with positive scale parameter α and positive shape parameter β. An extreme value random variable X with parameters α and β has probability density function

$$f(x) = (\beta/\alpha)e^{x\beta - e^{x\beta}/\alpha} \qquad -\infty < x < \infty.$$

The extreme value distribution can be used to model water levels associated with floods and earthquake magnitudes. The probability density function with three different parameter settings is illustrated below.



The cumulative distribution function is

$$F(x) = 1 - e^{-e^{\beta x}/\alpha} \qquad -\infty < x < \infty.$$

The survivor function on the support of X is

$$S(x) = P(X \geq x) = e^{-e^{\beta x}/\alpha} \qquad -\infty < x < \infty.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = \beta e^{\beta x}/\alpha \qquad -\infty < x < \infty.$$

The cumulative hazard function on the support of X is

$$H(x) = e^{\beta x}/\alpha \qquad -\infty < x < \infty.$$

The inverse distribution function of X is

$$F^{-1}(u) = \frac{\ln(-\alpha \ln(1-u))}{\beta} \qquad -\infty < x < \infty.$$

The population mean, variance, skewness, and kurtosis of X do not have closed-form expressions.

APPL verification: The APPL statements

```
X := ExtremeValueRV(alpha, beta);
CDF(X);
SF(X)
HF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

verify the survivor function, hazard function, and cumulative hazard function, and show that the population mean, variance, skewness, kurtosis, and moment generating function do not have closed-form expressions.