Exponential power distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)

The shorthand $X \sim \text{exponential power}(\lambda, \kappa)$ is used to indicate that the random variable $X$ has the exponential power distribution with positive scale parameter $\lambda$ and positive shape parameter $\kappa$. An exponential power random variable $X$ has probability density function

$$f(x) = \left( e^{1-e^{\lambda x \kappa}} \right) e^{\lambda x \kappa \lambda \kappa x^{\kappa-1}} \quad x > 0.$$ 

The exponential power distribution is one of the few two-parameter distributions that can achieve a bathtub-shaped hazard function. The probability density function for three different parameter settings is illustrated below.

![Exponential power density function](image)

The cumulative distribution function on the support of $X$ is

$$F(x) = P(X \leq x) = 1 - e^{1-e^{\lambda x \kappa}} \quad x > 0.$$ 

The survivor function on the support of $X$ is

$$S(x) = P(X \geq x) = e^{1-e^{\lambda x \kappa}} \quad x > 0.$$ 

The hazard function on the support of $X$ is

$$h(x) = \frac{f(x)}{S(x)} = e^{\lambda x \kappa \lambda \kappa x^{\kappa-1}} \quad x > 0.$$ 

The cumulative hazard function on the support of $X$ is

$$H(x) = -\ln S(x) = e^{\lambda x \kappa} - 1 \quad x > 0.$$ 

The inverse distribution function of $X$ is

$$F^{-1}(u) = \left[ \frac{1}{\lambda} \ln(1 - \ln(1 - u)) \right]^{1/\kappa} \quad 0 < u < 1.$$
The median of $X$ is

$$F^{-1}(1/2) = (\lambda \ln(1 - \ln(1/2)))^{-1/\kappa}.$$  

The moment generating function of $X$ is

$$M(t) = E[e^{tX}] = \int_{0}^{\infty} \lambda \kappa x^{\kappa-1} e^{(tx + 1 + \lambda x) - e^{\lambda x}} \, dx.$$  

The characteristic function of $X$ is

$$\phi(t) = E[e^{itX}] = \int_{0}^{\infty} \lambda \kappa x^{\kappa-1} e^{itx + 1 + \lambda x - e^{\lambda x}} \, dx.$$  

The population mean, variance, skewness, and kurtosis of $X$ are mathematically intractable.

**APPL verification:** The APPL statements

```appl
X := ExponentialPowerRV(lambda, kappa);
CDF(X);
SF (X);
HF (X);
IDF(X);
MGF(X);
```

verify the cumulative distribution function, hazard function, inverse distribution function, and moment generating function.