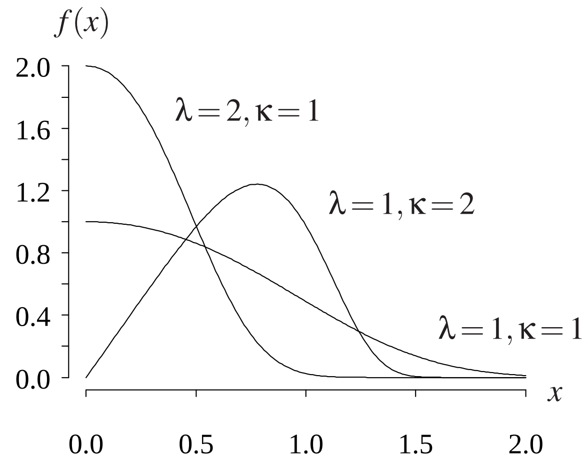
Exponential power distribution (from http://www.math.wm.edu/~leemis/chart/UDR/UDR.html)
The shorthand $X \sim$ exponential power(λ, κ) is used to indicate that the random variable X has the exponential power distribution with positive scale parameter λ and positive shape parameter κ. An exponential power random variable X has probability density function

$$f(x) = \left(e^{1-e^{\lambda x^{\kappa}}}\right) e^{\lambda x^{\kappa}} \lambda \kappa x^{\kappa-1} \qquad x > 0.$$

The exponential power distribution is one of the few two-parameter distributions that can achieve a bathtub-shaped hazard function. The probability density function for three different parameter settings is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \le x) = 1 - e^{1-e^{\lambda x^{\kappa}}} \qquad x > 0.$$

The survivor function on the support of X is

$$S(x) = P(X \ge x) = e^{1-e^{\lambda x^{\kappa}}} \qquad x > 0.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = e^{\lambda x^{\kappa}} \lambda \kappa x^{\kappa-1} \qquad x > 0.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = e^{\lambda x^{\kappa}} - 1 \qquad x > 0.$$

The inverse distribution function of X is

$$F^{-1}(u) = \left[\frac{1}{\lambda} \ln\left(1 - \ln(1-u)\right)\right]^{1/\kappa} \qquad 0 < u < 1.$$

The median of X is
$$F^{-1}(1/2) = (\lambda \ln(1 - \ln(1/2)))^{-1/\kappa}.$$
The moment generating function of X is
$$M(t) = E\left[e^{tX}\right] = \int_0^\infty \lambda\kappa x^{\kappa-1} e^{tx+1+\lambda x^\kappa - e^{(\lambda x^\kappa)}}\, dx.$$
The characteristic function of X is
$$\phi(t) = E\left[e^{itX}\right] = \int_0^\infty \lambda\kappa x^{\kappa-1} e^{itx+1+\lambda x^\kappa - e^{(\lambda x^\kappa)}}\, dx.$$
The population mean, variance, skewness, and kurtosis of X are mathematically intractable.

APPL verification: The APPL statements

```
X := ExponentialPowerRV(lambda, kappa);
CDF(X);
SF(X);
HF(X);
IDF(X);
MGF(X);
```

verify the cumulative distribution function, hazard function, inverse distribution function, and moment generating function.