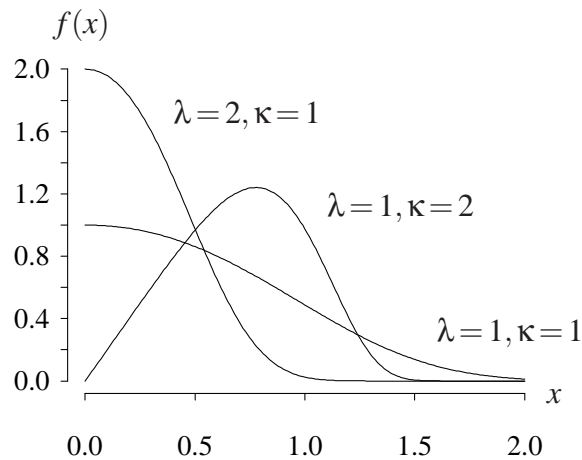


Exponential power distribution (from <http://www.math.wm.edu/~leemis/chart/UDR/UDR.html>)

The shorthand $X \sim \text{exponential power}(\lambda, \kappa)$ is used to indicate that the random variable X has the exponential power distribution with positive scale parameter λ and positive shape parameter κ . An exponential power random variable X has probability density function

$$f(x) = \left(e^{1-e^{\lambda x^\kappa}} \right) e^{\lambda x^\kappa} \lambda \kappa x^{\kappa-1} \quad x > 0.$$

The exponential power distribution is one of the few two-parameter distributions that can achieve a bathtub-shaped hazard function. The probability density function for three different parameter settings is illustrated below.



The cumulative distribution function on the support of X is

$$F(x) = P(X \leq x) = 1 - e^{1-e^{\lambda x^\kappa}} \quad x > 0.$$

The survivor function on the support of X is

$$S(x) = P(X \geq x) = e^{1-e^{\lambda x^\kappa}} \quad x > 0.$$

The hazard function on the support of X is

$$h(x) = \frac{f(x)}{S(x)} = e^{\lambda x^\kappa} \lambda \kappa x^{\kappa-1} \quad x > 0.$$

The cumulative hazard function on the support of X is

$$H(x) = -\ln S(x) = e^{\lambda x^\kappa} - 1 \quad x > 0.$$

The inverse distribution function of X is

$$F^{-1}(u) = (\lambda \ln(1 - \ln(1 - u)))^{-1/\kappa} \quad 0 < u < 1.$$

The median of X is

$$F^{-1}(1/2) = (\lambda \ln(1 - \ln(1/2)))^{-1/\kappa}.$$

The moment generating function of X is

$$M(t) = E[e^{tX}] = \int_0^{\infty} \lambda \kappa x^{\kappa-1} e^{tx+1+\lambda x^{\kappa}-e^{(\lambda x^{\kappa})}} dx.$$

The characteristic function of X is

$$\phi(t) = E[e^{itX}] = \int_0^{\infty} \lambda \kappa x^{\kappa-1} e^{itx+1+\lambda x^{\kappa}-e^{(\lambda x^{\kappa})}} dx.$$

The population mean, variance, skewness, and kurtosis of X are mathematically intractable.

APPL verification: The APPL statements

```
X := ExponentialPowerRV(lambda, kappa);  
CDF(X);  
SF(X);  
HF(X);  
IDF(X);  
MGF(X);
```

verify the cumulative distribution function, hazard function, inverse distribution function, and moment generating function.