Theorem The exponential distribution has the scaling property. That is, if X is an exponential random variable with population mean $\alpha > 0$, then for any constant $k > 0$, kX is also an exponential random variable.

Proof The cumulative distribution function of an exponential random variables X is

$$\begin{aligned} F_X(x) &= P(X \le x) \\ &= 1 - e^{-x/\alpha} \qquad\qquad x > 0. \end{aligned}$$

The cumulative distribution function of the random variable kX, for $k > 0$ is

$$\begin{aligned} F_{kX}(x) &= P(kX \le x) \\ &= P(X \le x/k) \\ &= 1 - e^{-x/(k\alpha)} \qquad\qquad x > 0, \end{aligned}$$

which is also the cumulative distribution function of an exponential random variable. Therefore, the exponential distribution has the scaling property.

APPL verification: The APPL statement

```
assume(alpha > 0);
assume(k > 0);
X := [[x -> exp(-x / alpha) / alpha], [0, infinity], ["Continuous", "PDF"]];
g := [[x -> k * x], [0, infinity]];
Y := Transform(X, g);
```

yields the probability density function of an exponential random variable, verifying the scaling property of the exponential distribution.