Theorem If $X_i \sim$ exponential(λ_i), for $i = 1, 2, \ldots, n$, and $X_1, X_2, \ldots, X_n$ are mutually independent random variables, then

$$\min\{X_1, X_2, \ldots, X_n\} \sim \text{exponential}\left(\sum_{i=1}^{n} \lambda_i\right).$$

Proof The random variable X_i has cumulative distribution function

$$F_{X_i}(x) = P(X_i \le x) = 1 - e^{-\lambda_i x_i} \qquad\qquad x > 0$$

for $i = 1, 2, \ldots, n$. Let the random variable $Y = \min\{X_1, X_2, \ldots, X_n\}$. Then the cumulative distribution function of Y is

$$\begin{aligned}
F_Y(y) &= P(Y \le y) \\
&= 1 - P(Y \ge y) \\
&= 1 - P\left(\min\{X_1, X_2, \ldots, X_n\} \ge y\right) \\
&= 1 - P\left(X_1 \ge y, X_2 \ge y, \ldots, X_n \ge y\right) \\
&= 1 - P\left(X_1 \ge y\right) P\left(X_2 \ge y\right) \ldots P\left(X_n \ge y\right) \\
&= 1 - e^{-\lambda_1 y} e^{-\lambda_2 y} \ldots e^{-\lambda_n y} \\
&= 1 - e^{-\lambda_1 y - \lambda_2 y - \cdots - \lambda_n y} \\
&= 1 - e^{-\sum_{i=1}^{n} \lambda_i y} \qquad\qquad y > 0.
\end{aligned}$$

This cumulative distribution function can be recognized as that of an exponential random variable with parameter $\sum_{i=1}^{n} \lambda_i$.

APPL illustration: The APPL statements to find the probability density function of the minimum of an exponential(λ_1) random variable and an exponential(λ_2) random variable are:

```
X1 := ExponentialRV(lambda1);
X2 := ExponentialRV(lambda2);
Minimum(X1, X2);
```

These statements yield an exponential distribution for the minimum with parameter $\lambda_1 + \lambda_2$.