Theorem If $X_i \sim$ exponential(λ_i) for $i = 1, 2, \ldots, n$ are mutually independent random variables with survivor functions

$$S_i(x) = e^{-\lambda_i x} \qquad x > 0,$$

for $i = 1, 2, \ldots, n$, the positive constants $p_1, p_2, \ldots, p_n$ satisfy $\sum_{i=1}^{n} p_i = 1$, and one of the exponential distributions is chosen according to the probabilities $p_1, p_2, \ldots, p_n$, and the corresponding exponential random variable X is observed, then the unconditional survivor function is

$$S_X(x) = \sum_{i=1}^{n} p_i e^{-\lambda_i x} \qquad x > 0.$$

Proof The conditional survivor function based on the ith exponential distribution being chosen is

$$S_i(x) = e^{-\lambda_i x} \qquad x > 0$$

for some i in the range $1, 2, \ldots, n$. Let X be the unconditional mixture random variable as described above. The unconditional survivor function of X is a weighted average of the conditional survivor functions, e.g.,

$$S_X(x) = \sum_{i=1}^{n} p_i e^{-\lambda_i x} \qquad x > 0.$$

Maple illustration: The APPL statements

```
X1 := ExponentialRV(1);
X2 := ExponentialRV(2);
X3 := ExponentialRV(3);
p1 := 7 / 10;
p2 := 2 / 10;
p3 := 1 / 10;
X  := Mixture([p1, p2, p3], [X1, X2, X3]);
```

yield the probability density function of the mixture

$$f_X(x) = \frac{7}{10} e^{-x} + \frac{2}{5} e^{-2x} + \frac{3}{10} e^{-3x} \qquad x > 0.$$