Theorem The error distribution has the scaling property. That is, if $X \sim \text{error}(a, b, c)$ then $Y = kX$ also has the error distribution.

Proof Let the random variable X have the error(a, b, c) distribution with probability density function

$$f(x) = \frac{e^{(-|x-a|/b)^{2/c}/2}}{b(2^{c/2+1})\Gamma\left(c/2+1\right)} \qquad\qquad -\infty < x < \infty.$$

Let k be a positive, real constant. The transformation $Y = g(X) = kX$ is a 1–1 transformation from $\mathcal{X} = \{x \mid -\infty < x < \infty\}$ to $\mathcal{Y} = \{y \mid -\infty < y < \infty\}$ with inverse $X = g^{-1}(Y) = Y/k$ and Jacobian

$$\frac{dX}{dY} = \frac{1}{k}.$$

Therefore, by the transformation technique, the probability density function of Y is

$$\begin{aligned} f_Y(y) &= f_X(g^{-1}(y))\left|\frac{dx}{dy}\right| \\ &= \frac{e^{(-|y/k-a|/b)^{2/c}/2}}{b(2^{c/2+1})\Gamma\left(c/2+1\right)}\left|\frac{1}{k}\right| \\ &= \frac{e^{(-|y-ka|/kb)^{2/c}/2}}{kb(2^{c/2+1})\Gamma\left(c/2+1\right)} \qquad\qquad -\infty < x < \infty, \end{aligned}$$

which is the probability density function of an error(ka, kb, c) random variable.

APPL failure: The APPL statements

```
assume(k > 0);
X := ErrorRV(a, b, c);
g := [[x -> k * x], [0, infinity]];
Y := Transform(X, g);
```

fail to produce the probability density function of an error(ka, kb, c) random variable.