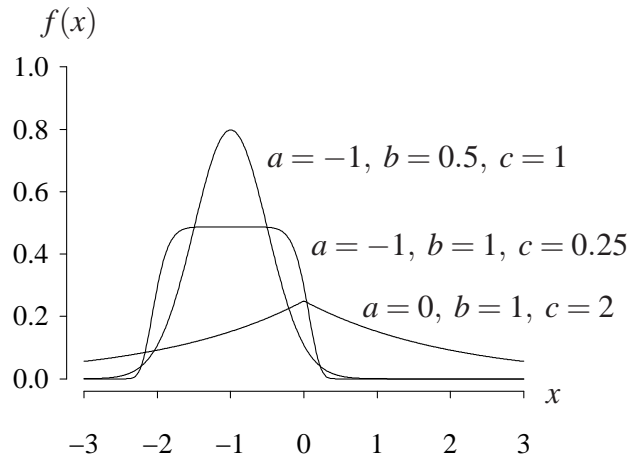


**Error distribution** (from <http://www.math.wm.edu/~leemis/chart/UDR/UDR.html>)

The shorthand  $X \sim \text{error}(a, b, c)$  is used to indicate that the random variable  $X$  has the error distribution with location parameter  $a$ , scale parameter  $b$ , and shape parameter  $c$ . An error random variable  $X$  with parameters  $a$ ,  $b$ , and  $c$  has probability density function

$$f(x) = \frac{e^{(-|x-a|/b)^{2/c}/2}}{b(2^{c/2+1})\Gamma(c/2+1)} \quad -\infty < x < \infty$$

for all real values  $a$  and for  $b > 0$ ,  $c > 0$ . The probability density function with three different parameterizations is illustrated below.



The cumulative distribution function on the support of  $X$  is

$$F(x) = P(X \leq x) = \int_{-\infty}^x \frac{e^{(-|t-a|/b)^{2/c}/2}}{b(2^{c/2+1})\Gamma(c/2+1)} dt \quad -\infty < x < \infty.$$

The survivor function, hazard function, inverse distribution function, moment generating function, and characteristic functions are all mathematically intractable. The population median and mode of  $X$  occurs at  $x = a$ . The population mean, variance, skewness, and kurtosis of  $X$  are

$$E[X] = a \quad V[X] = \frac{2^c b^2 \Gamma(3c/2)}{\Gamma(c/2)}.$$

$$E \left[ \left( \frac{X - \mu}{\sigma} \right)^3 \right] = 0 \quad E \left[ \left( \frac{X - \mu}{\sigma} \right)^4 \right] = \frac{\Gamma(5c/2)\Gamma(c/2)}{(\Gamma(3c/2))^2}.$$

**APPL verification:** The APPL statements

```
assume(b > 0);
assume(c > 0);
X := [[x -> exp((-abs(x - a) / b) ^ (2 / c) / 2) / (b * (2 ^ (c / 2 + 1)) *
      GAMMA(c / 2 + 1))], [-infinity, infinity], ["Continuous", "PDF"]];
CDF(X);
SF(X);
HF(X);
CHF(X);
Mean(X);
Variance(X);
Skewness(X);
Kurtosis(X);
MGF(X);
```

fail due to integration problems.