Theorem The exponential distribution is a special case of the Erlang distribution when $n = 1$.

Proof Let the random variable $X \sim \text{Erlang}(\alpha, n)$. The probability density function of X is

$$f(x) = \frac{x^{n-1} e^{-x/\alpha}}{\alpha^n (n-1)!} \qquad\qquad x > 0.$$

Substituting $n = 1$ yields

$$\begin{aligned} f_X(x) &= \frac{x^{1-1} e^{-x/\alpha}}{\alpha^1 (1-1)!} \\ &= (1/\alpha) e^{-x/\alpha} \qquad\qquad x > 0, \end{aligned}$$

which is the probability density function of an exponential(α) random variable.

APPL verification: The APPL statements

```
n := 1;
X := ErlangRV(alpha, n);
Y := ExponentialRV(alpha);
```

confirm the result.